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CURRENT PAPERS AND DISCUSSIONS

		Discussion closes
Report of the Committee of Engineering Foundation on Arch Dam Investigation	May, 1928	
Discussion . . . Nov., 1928, Jan., April, May, Aug., 1929, May, 1930,	Mar., 1931	Closed
Effect of Turbulence on the Registration of Current Meters. *David L. Yarnell and Floyd A. Nagler*	Dec., 1929	
Discussion (Authors' closure). Feb., May, Aug., Sept., Nov., 1930, Mar.,	May, 1931	Closed
Analysis of Continuous Frames by Distributing Fixed-End Moments. *Hardy Cross*	May, 1930	
Discussion . . . Sept., Oct., Nov., 1930, Jan., Feb., Mar.,	May, 1931	Uncertain
Highways as Elements of Transportation. *Fred Lavis*	Aug., 1930	
Discussion (Author's closure) . . . Oct., Dec., 1930.	May, 1931	Closed
Some Aspects of Water Conservation. *R. A. Sutherland*	Sept., 1930	
Discussion . . . Jan., Feb.,	Mar., 1931	Closed
Construction of La Ola Pipe Line in Chile. *W. B. Saunders*	Sept., 1930	
Discussion	Feb., 1931	Aug., 1931
Tests of Broad-Crested Weirs. *James G. Woodburn*	Sept., 1930	
Discussion . . . Oct., Dec., 1930,	Feb., 1931	Aug., 1931
Stresses in Gravity Dams by Principle of Least Work. *B. F. Jakobsen*	Sept., 1930	
Discussion . . . Sept., Nov., 1930, Jan., Feb., Apr.,	May, 1931	Aug., 1931
Theory of Similarity and Models. *Benjamin F. Groat*	Oct., 1930	
Discussion . . . Jan., Feb.,	Mar., 1931	Aug., 1931
Formulas for Rainfall Intensities of Long Duration. *Merrill M. Bernard.*	Oct., 1930	
Discussion . . . Feb.,	May, 1931	Aug., 1931
The Buttressed Dam of Uniform Strength. *Herman Schorer.*	Nov., 1930	
Discussion . . . Jan., Feb., Mar.,	May, 1931	Aug., 1931
Economical Design of Hydraulic Pipe-Line Dredge. *John Francis Cushing.*	Nov., 1930	
Discussion . . . Feb.,	Mar., 1931	Aug., 1931
Highway Location: General Considerations. *Gibb Gilchrist*	Nov., 1930	
Discussion	Feb., 1931	Aug., 1931
Highway Location: Practical Considerations. *A. R. Losh*	Nov., 1930	
Discussion . . . Feb.,	May, 1931	Aug., 1931
Relocating Land Surveys in Texas. *B. F. Williams*	Nov., 1930	
Discussion	Apr., 1931	Aug., 1931
Flow of Water in Tidal Canals. *Earl I. Brown*	Dec., 1930	
Discussion . . . Feb., Mar.,	May, 1931	Sept., 1931
The Don Martin Project. *Andrew Weiss*	Dec., 1930	
Discussion . . . Mar.,	May, 1931	Sept., 1931
Design of a Reinforced Concrete Skew Arch. *Bernard L. Weiner.*	Jan., 1931	
Discussion	May, 1931	Sept., 1931
A Discharge Diagram for Uniform Flow in Open Channels. *Murray Blanchard*	Jan., 1931	
Discussion	May, 1931	Sept., 1931
Legal Problems Involved in Establishing Set-Back Lines. *Clifton Williams.*	Jan., 1931	
Discussion	May, 1931	Sept., 1931
The Eight-Mile Cascade Tunnel, Great Northern Railway: A Symposium.	Feb., 1931	Sept., 1931
Measuring Irrigation Deliveries in the Panjab. *E. S. Lindley*	Feb., 1931	Sept., 1931
Surveys for Swift River Reservoir of the Boston Metropolitan Water Supply. *N. LeRoy Hammond*	Feb., 1931	
Discussion	May, 1931	Sept., 1931
Flow of Ground-Water as Applied to Drainage Wells. *M. R. Lewis*	Mar., 1931	Sept., 1931
Gravity Dams Arched Down Stream. *B. J. Lambert*	Mar., 1931	Sept., 1931
An Analysis of Multiple-Skew Arches on Elastic Piers. *J. Charles Rathbun*	Apr., 1931	Sept., 1931
Run-off—Rational Run-off Formulas. *R. L. Gregory and C. E. Arnold*	Apr., 1931	Sept., 1931

CONTENTS FOR MAY, 1931

PAPERS

PAGE

Manufacturing Concrete of Uniform Quality.
By William M. Hall, M. Am. Soc. C. E. 675

REPORTS OF COMMITTEES

Specifications for Boundary Surveys.
Prepared by the Committee of the Surveying and Mapping Division on Boundary Surveys. 699

DISCUSSIONS

Effect of Turbulence on the Registration of Current Meters.
By Messrs. Robert E. Horton, Richard Van Vliet, and David L. Yarnell and Floyd A. Nagler. 703

Analysis of Continuous Frames by Distributing Fixed-End Moments.
By Messrs. T. F. Hickerson, F. H. Constant, W. N. Downey, and E. C. Hartmann. 725

Highways as Elements of Transportation.
By Messrs. Edwin F. Wendt, H. W. Hudson, and Fred Lavis. 747

Stresses in Gravity Dams by Principle of Least Work.
By Messrs. Fernand Campus, A. Floris, Robert E. Glover, and P. Wilhelm Werner. 753

Formulas for Rainfall Intensities of Long-Duration.
By Messrs. R. W. Powell, C. E. Grunsky, and Charles W. Sherman. . . . 771

The Buttressed Dam of Uniform Strength.
By Messrs. Eugene Kalman, A. Floris, Fredrik Vogt, and A. C. Schwager. 779

Highway Location: Practical Considerations.
By Messrs. J. C. Carpenter, H. J. Spelman, H. S. Kerr, and H. W. Giffin. 793

Flow of Water in Tidal Canals.
By H. F. Flynn, M. Am. Soc. C. E. 801

The Don Martin Project.
By Robert S. Stockton, M. Am. Soc. C. E. 807

Design of a Reinforced Concrete Skew Arch.
By Messrs. G. D. Houtman, and J. Charles Rathbun. 809

CONTENTS FOR MAY, 1931 *(Continued)*

A Discharge Diagram for Uniform Flow in Open Channels.
By Messrs. Lynn Perry, J. B. MacPhail, G. B. Pillsbury, Lynn Crandall, and Benjamin E. Jones. 817

Legal Problems Involved in Establishing Set-Back Lines.
By Messrs. J. M. Albers, J. E. Willoughby, Horace H. Sears, George H. Herrold, Clifford C. Muhs, H. B. Cooley, Walter C. Sadler, H. J. McFarlan, Donald M. Baker, and E. G. Walker. 825

Surveys for Swift River Reservoir of the Boston Metropolitan Water Supply.
By Messrs. C. B. Breed, and C. L. Hall. 845

For Index to all Papers, the discussion of which is current in* PROCEEDINGS, *see page 2

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MEMBERSHIP

Applications for Admission and Transfer. . . . . following page 850

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

MANUFACTURING CONCRETE OF UNIFORM QUALITY

By William M. Hall,[1] M. Am. Soc. C. E.

Synopsis

Most of the literature on monolithic concrete construction appears to be devoted to laboratory practice, the water-cement ratio theory, and subjects relating thereto. Although the knowledge and technique thus obtained are essential for the construction engineer, they may not be as difficult to acquire, as it is to perfect the field methods of mixing and placing concrete of a desired uniform quality and strength.

In 1930, and a few years prior thereto, the United States Government completed, under direction of Lt. Col. George R. Spalding, U. S. A., nine locks and dams in the Ohio River between Louisville, Ky., and its mouth (nearly 400 miles). This task required more than 750 000 cu. yd. of concrete. In 1925, a systematic and uniform method of inspection and control of concrete manufacture was authorized, inaugurated for all the dams, and carried through to the end of the work. The purpose of this paper is to record the results obtained and to give a detailed description of the methods pursued with the hope of inspiring in others an ambition to adopt and perfect the practice of manufacturing concrete of a specified strength and grade with a reliability equal to that of the best of other structural materials.

With a realization of the importance of the perfect execution of all the inspection and of the co-operation of all the working forces in every detail of field work required in the manufacture and placing of concrete and in the fair sampling and making, curing, and breaking of cylinders, the writer has included in Appendices a detailed description of the methods followed by two field inspectors, Messrs. R. W. McBeth and C. F. Little.

The nine locks and movable dams are the last of a series of fifty built for the improvement of the Ohio River so that with slack-water there will be

Note.—Written discussion on this paper will be closed in September, 1931, *Proceedings*.

[1] U. S. Civ. Engr., Parkersburg, W. Va.

a minimum depth of 9 ft. on the lock-gate sills and the navigable pass sills of the dams. All the structures are now (1931) in operation at a total cost of construction to the United States of about $115 000 000.

Introduction

The requirement of first importance for any well-designed concrete structure is that the concrete be satisfactory as to: (1) Soundness; (2) density; and (3) uniform strength, approximately as designed, throughout every part of the structure. Although due regard should be given to the element of cost, every concrete structure should have a probable life as long as its intended use, or if that is unknown, for a period of more than 100 years, except in the parts which may be worn out by severe abrasion.

Although it is now nearly forty years since, in the early Nineties, the design and construction of monolithic concrete structures became active in the United States and Europe, more than 50% of the concrete being made is of poor quality. Its probable life is short and only a small part will comply with conditions for quality as proposed for it. Therefore, the question of how to eliminate the causes of such failures has long been of great importance. Of the three major qualities sought in its manufacture, uniform strength is usually the one of greatest concern; or, expressed otherwise, the improvement most desired is to design a mix of minimum required strength and, then, with confidence, to be able to guarantee every batch of it.

Although the choice of materials is of paramount importance, the personal element is a large factor in the manufacturing and testing of the concrete. Variations in quality may occur due to: (1) The method of procuring, inspecting, and delivering the materials; (2) the choice and installation of the mixer; (3) the arrangement of all the features of transportation from mixer to the finished concrete in the form; (4) the measurement of all the materials entering the mixer; (5) the method of charging; and (6) the sampling, making, seasoning, and breaking of test cylinders. Not only do the inspection and cylinder records indicate the relative durability and value of concrete, but so also does its appearance when being placed and when finished. When inspection records are favorable with regard to these features the engineer, architect, and owner may feel confident of the safety and durability of the structure, provided its placement is known to be well and thoroughly done.

It is with this thought and for these reasons that the writer is submitting some records of work done, methods pursued, and experience gained in an effort to produce concrete of uniform quality. No one or two easy remedies can be formulated or found, but the way to success, like that which has been required for all other good construction, will include the systematic training of the workmen who are entrusted with the principal details of concrete manufacture, placing, and treatment. Uniform success in producing good concrete requires reasonably suitable materials; and it also requires contractors, superintendents, and foremen with the spirit to co-operate; competent inspectors; and workmen with training, skill, and reliability for measuring quantities of

water, cement, and aggregates, day after day, with unfailing accuracy. It is not intended to discuss theories, of concrete design, laboratory practice, or specifications, except as they affect the inspector's work in the field. This task involves the selecting and inspecting of materials and the teaching and directing of the workmen as to their duties and faithful performance. By excellency of control he may be able to make the contractor and other managers of the work realize that such methods are for their best interest in producing concrete with a favorable record for quality, and that the net result will be economy in the cost of the finished work.

The greatest reason for much defective work is due to the owner's ignorance of its quality (or of how badly he has been duped), immediately after completion. Many engineers in highest authority have accepted a poor quality of concrete only to realize it years later. The appearance of the mix or an immediate test of the concrete, when freshly mixed or placed, does not disclose its quality with sufficient accuracy to separate the poor from the good, except that it indicates when excess water is present, that aggregate has segregated, or that the mix was greatly deficient in cement, sand, or water. Furthermore, the need of either accepting the concrete immediately or of condemning and destroying it has tended to permit the construction of much poor work. The cost of repairs or early renewals of defective concrete constructed in the United States to date (1931) will probably be in the tens of millions of dollars.

Although in the Nineties monolithic concrete construction was strongly condemned by engineers of high repute as an unreliable structural material, the great urgency for speed in the construction of fortifications in 1898, at the twenty-six strategic points encircling the coast line of the United States from Maine to Oregon, so broadcast the knowledge of its adaptability for rapid masonry construction that its use in all classes of bridge and industrial masonry was thereby hastened. Since then its use has been extended rapidly to include almost all classes of masonry. It was possibly the great extent of that fortification construction program and the concurrent demand for speed that saved it from general condemnation as a fit structural material and substitute for stone and brick masonry. The great pity is that in the decade following 1895, some super-power could not have controverted entirely the arguments advanced by advocates of excessively wet concrete mixtures and thus have avoided the introduction of long chutes which were so injurious, or ruinous, to good concrete. These two wrong methods of construction and a few others, which were much in use from 1900 to 1920, were apparently in danger of eliminating almost entirely the use of the better methods of mixing and placing which were very well known and strongly advocated by a few.

Practice on the Ohio River

Most papers and published records show an almost complete lack of uniformity in terms of maintained strength for job concrete up to or above that designed, even with aggregates of fairly uniform quality of grading and of water content. For many years on the construction of the Ohio River locks and dams, sand and gravel (usually with poor natural grading), were dug at

the work and delivered at the mixer for immediate use. This practice made the control of the water content much more difficult than with stored aggregates. Many efforts were made to secure better concrete under such difficult conditions, especially in the past five years (1926-1931) during which nearly 750 000 cu. yd. of monolithic concrete were placed in the nine locks and movable dams constructed in the Louisville District of the Ohio River. A brief statement of the methods used and the results obtained may contribute to a better understanding of the requirements necessary in organizing and conducting all the most essential details of mixing, placing, and testing concrete.

The concrete construction of Auxiliary Lock 41, at Louisville, Dam 47, near Evansville, Ind., Lock 51 and Dam 51 at Golconda, Ill., was done by contract, and the remainder by hired labor forces under the direction of a resident "Assistant in Charge", reporting directly to the District Engineer. The location and general description of these structures have been discussed by the writer in another paper.[2]

The Concrete Inspectors.—The inspectors assigned to mixing and placing, and the cylinder makers, were part of a force organized in 1925 for the field work of laying out and inspecting the work of construction of the locks and dams. Many members of the assembled force and nearly all the men at the mixers had previously had very little experience, instruction, or laboratory and field training in manufacturing or placing concrete. This accounts for the large number of failing cylinders shown in the report for 1925.

At the beginning of the work, all the members of the concrete inspection force were required to study a digest of the literature on the subject, including the theory of Duff A. Abrams, M. Am. Soc. C. E., on the water-cement-ratio, his proposed methods of determining fineness modulus for aggregates, and other methods of control. A system of making and testing concrete cylinders was also begun in August, 1925, A short time afterward the inspection forces, with some of the principal members of the construction force, were organized as a school. Its first meeting of a week, with two sessions daily, was held in January, 1926, at Dam 52, Brookport, Ill., the next two sessions were held at Owensboro, Ky., and the fourth was held in Louisville, in February, 1928. Under a leader, the inspectors were required to study their respective stations, the specifications for the job, the specifications of the American Society for Testing Materials, and the several bulletins[3] and booklets[4] formulated by Professor Abrams. Then they attended classes, as stated previously, conducted by the writer, with J. W. Kelly, M. Am. Soc. C. E. (through the courtesy of the Portland Cement Association) as principal teacher. Papers were also prepared, read, and discussed by members of the inspection force on questions relating to the plant, their own difficulties, and construction features at the various locks and dams. Copies of some of the complete papers, and carefully prepared minutes of all sessions, were later furnished to the entire force. Under the Abrams' standard for average

[2] *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 94, Fig. 1.

[3] *Bulletins 1* to *12*, Lewis Inst., Structural Materials Research Laboratories, Chicago, Ill.

[4] Pub. by the American Portland Cement Assoc.

strength of cement,[5] the specifications for the job should produce a concrete made with the average Ohio River gravel, with a compressive strength of about 1 700 lb. per sq. in. at 28 days. The job standard of minimum strength was set at 2 000 lb. per sq. in. for 28-day concrete, and 2 800 lb. per sq. in. at 6 months which is a high standard for the specified proportion of cement to aggregate.

During 1925, 1926, and 1927, only one cylinder was made from each sample or batch of concrete. In 1928 and 1929, the cylinders were made in pairs, two cylinders from each sample or batch tested. In this manner one cylinder of the pair checked the fabrication of the other. If one cylinder failed in proposed strength or if the tests showed a great difference in strengths, it was believed that the higher record could be assumed to be the true measure of quality of the entire batch from which the sample was taken. Throughout the seasons of 1925 to 1928, inclusive, the cylinders were packed in sand, two in a box, and shipped to a laboratory at Louisville. During 1929, the cylinders were broken under contract at the Testing Laboratory of the University of Louisville. The agreement included the care of the cylinders during the last days of seasoning, and re-capping them when the ends were not perfect. The record for 1929 included every cylinder made. For the other years only about half a dozen cylinders were rejected due to injury or other cause that clearly made the break recorded an unfair test. Table 1 indicates the strength of the concrete. Of the thirty-eight cylinders (nineteen pairs) made at Auxiliary Lock 41, Louisville, which were tested at the age of six months (see Column (12), Table 1), four (each cylinder of two of the pairs) were subjected to 200 000 lb. (7 100 lb. per sq. in.) and failed to break. This was the limit of compression for the testing machine available. One month later, one of these cylinders broke, in the same machine, under a total pressure of 194 000 lb., and its mate at 182 820 lb., or 6 450 lb. per sq. in.

Specifications

The specifications for the work provided that the concrete should be composed of cement, hydrated lime, sand, gravel, and water. The cement measure was by count of bags for each batch. The aggregates were measured by leveling up to exact marks set in the charging hoppers or by batchers and inundators. The water was measured by gauges in barrels or by water meters. The sand, gravel, and water were taken from the Ohio River, under specifications requiring the best that could be produced in the locality of the structure. The mix per cubic yard of concrete consisted of 5 bags of cement (6 bags for all reinforced concrete and tops of walls), 25 lb. of hydrated lime, or other equally good admixture; and 25 cu. ft. of gravel. Sand was required sufficient to yield 15% or more of mortar than was needed to fill the voids in the gravel and sufficient water to produce a slump of 2 to 4 in. for mass concrete and 5 to 6 in. for reinforced concrete; that is, such that the feet would not sink in freshly spread concrete deeper than 10 in., nor less than 2 in.

[5] *Bulletin 1*, "Design of Concrete Mixtures," by Duff A. Abrams, Lewis Inst., Structural Materials Research Laboratory, Chicago, Ill.

TABLE 1.—CYLINDER TESTS OF CONCRETE IN DAMS ON THE OHIO RIVER IMPROVEMENT

Item	SEASONS:	1925		1926		1927		1928		1929	
	Age:	28 days	6 months	28 days	6 months	28 days	6 months	28 days	6 months	28 days	6 months
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1	Number of cylinders..	81	44	172	90	236	134	206	109	64	40
2	Average strength per cylinder, in pounds per square inch.....	2 157	3 046	2 719	3 777	2 685	4 119	2 691	3 640	4 779	6 082
3	Average strength of all cylinders testing less than Item 2, in pounds per square inch......	1 590	2 321	2 186	3 047	2 128	3 394	2 215	3 194	4 308	5 529
4	Average strength of all cylinders testing more than Item 2, in pounds per square inch..............	2 933	3 720	3 217	4 444	3 302	4 801	3 326	4 130	5 186	6 587
5	Percentage of cylinders testing less than Item 2.............	58.3	42.2	48.8	47.8	52.5	48.5	56.3	53.2	62.5	47.5
6	Percentage of cylinders testing less than 2 000 lb. per sq. in....	56.8	......	11.6	......	15.2	......	11.7	......	0	......
7	Percentage of cylinders testing less than 2 800 lb. per sq. in....	......	37.7	......	13.3	......	4.5	......	7.3	......	0
	Compressive strength, in pounds per square inch:										
8	Maximum.........	5 256	4 725	4 424	5 358	4 512	5 836	5 120	5 210	5 930	7 000+
9	Minimum..........	919	1 592	920	1 592	1 027	2 300	1 260	2 260	3 417	4 790
	Slump, in inches:										
10	Average............	3.24	3.3	4.04	13.8	3.6	3.5	3.24	3.5	3.09	3.0
11	Maximum...........	7½	......	7	......	7	......	8½	......	6	4.5
12	Minimum............	½	......	1	......	1	......	½	......	2	1.5
13	Number of tests in which the slump was less than in Item 10..	49	19	91	53	130	63	111	59	40	14

It is believed that the workmen complied with these specifications in general, except for the batches that fell below the desired strength, or those below the proposed strength (see Items 6 and 7, Table 1). In these cases there was probably either some defect in complying with the specifications or in the making, handling, or breaking of cylinders that were defective. It was the rule to keep a complete record of every detail which might affect the strength of every cylinder from the time the sample was taken to the time the cylinder was broken. Even so, in the case of more than 50% of failures to exceed the proposed strength, it was impossible to find an adequate explanation of the cause. The observable cause of greatest frequency was the segregation of large-sized gravel which produced evident weakness because the mortar peeled off and left a clean, bright surface of stone. Another frequent condition observed was a lengthwise split in the cylinder, indicating an imperfect bearing on the end. In 1929, this defect was eliminated by greater care in detecting and re-capping slightly defective ends.

INSPECTION

The cost of maintaining any desired minimum strength will probably not be any more than that of concrete which actually fluctuated much above and

below such standard of strength. Uniform maintenance certainly requires a knowledge of concrete materials and of manufacture on the part of all concerned and also a systematic and constant control, as may be better understood from Table 1 and a few comments based on the instructions given the inspection force from time to time.

Advance Study.—Before being placed on duty, inspectors were instructed and helped to become thoroughly familiar with the specifications, the tests relating thereto, and the methods of making them. Furthermore, they were taught the duties involved in the direction and instruction of workmen. They were also instructed in the technique of obtaining samples of materials and of analyzing the different grades of sand and coarse aggregates available.

Laboratory Experience.—For a better understanding of the literature studied, each inspector analyzed the sand and gravel being used. He mixed concrete by hand and made experimental cylinders from them, recording his results and reporting them in writing to his Senior Inspector. For each of the proposed slumps (approximately 2, 3, 4, 5, and 6 in. with the materials furnished and accepted for concrete on the particular job assigned), the inspector determined the water-cement-ratio and kept a record of it. Aggregates were analyzed and, if the particular job received several distinct grades of aggregates, tests were made of each. As soon as possible after construction was in progress, the inspector plotted "typical job curves" of 28-day cylinders, indicating the water-cement-ratio on each curve. Copies of such data were promptly forwarded to the office for review.

Practical Field Inspection.—The greatest possible care was required of inspectors in the field to obtain exact measurements of materials for each batch of concrete. The quantity of aggregates required by specification was their measure, "dry and rodded". Because the bulking of gravel is so slight and so nearly constant, its measure was considered exact or, at least, correct within 1 per cent. The measure of sand was variable because the gravel carried sand, and when an inundator was not used, the bulking was irregular. As nearly as possible, the measure of water was maintained so as to give a 3-in. slump, except that a 4 to 5-in. slump was required in case of concrete used with reinforcement and where it was difficult to place dry concrete. To maintain this slump uniformly every mixer inspector needed to understand the principles of the water-cement-ratio theory, the best and quickest methods of keeping informed as to measures of materials, quantity of water in aggregates, and method of correcting error in slump by changing the measure of water or sand, or cement in an emergency, and all things relating thereto at his plant. The slump and workability are functions of all the foregoing variables, and the importance of maintaining their uniformity was impressed on the inspectors. Before starting a monolith and at all times when the mixer was in operation the inspectors were expected to have perfect arrangements made for quick communication and co-operation, and for instant action.

As early as practicable, and before beginning the concrete mixing, the inspectors became thoroughly familiar with the operation of the mixing plant and with the personnel assigned to it; and they satisfied themselves

that every one assigned to a particular task could perform his duties accurately and efficiently. Furthermore, they inspected the form to see that it was completely finished with a well prepared foundation bed properly unwatered. If it was a connection with old concrete, they looked for laitance, and directed its thorough removal. The old surface, thus prepared, was completely covered with a thick layer of mortar or grout.

The regulation of the quantities of materials and of the time of turning the mixer for each batch of concrete are the mixer inspector's most important duties. When the mixer is in operation he should remain on duty until relieved by another inspector. Dumping or chuting concrete into forms should be done so as to require spreading only, or a short movement, at most, not exceeding 10 ft. Great care should be taken to prevent any part of the concrete from striking the forms, the forming rods, or the bracing timbers. These items were emphasized in the instructions issued to inspectors.

The greatest care possible was taken to obtain fair samples for slump tests, and those for cylinders were preferably taken from a 5-bag mix. The inspector was instructed to indicate on the records whether lime or an extra half bag of cement was used, or whether any other variation or irregularity occurred. The age, make, and bin number of cement, and the seasoning temperature of cylinders and the maker's name, were also required.

Regarding slump tests, the instructions specified that: Samples of concrete for slump tests shall be taken within 10 min. of its discharge from the mixer. When starting concrete work, the slump test shall be made from the first batches mixed, and continued until a 3-in. slump is obtained and is running uniform; thereafter, while mixing is in progress, it shall be made at intervals of 1 hour, or oftener in case the consistency appears to change. Samples for cylinders shall be taken from forms immediately after being dumped and within 10 min. of discharge from mixer. A chart similar to progress drawings shall be kept, indicating (1) the location of each concrete monolith; (2) the cylinder batches in each monolith by serial number; and (3) its approximate location in the monolith.

Instructions as to Removal of Forms.—No forms were permitted to be removed in less than the time stated in the specifications without actual physical inspection by an assistant in charge to determine the set of the concrete and without concurrence by the senior inspector. During summer weather it was considered entirely safe to remove bulkhead forms after 24 hours from lifts of approximately 6 ft.

During the summer, the inspectors were permitted to allow a few men to walk on the concrete for the purpose of brushing after about 6 hours, but no succeeding lifts of concrete could be cast above one lift for a period of 48 hours.

Especial attention to the curing of concrete was required, and inspectors were instructed to follow in detail the specifications as to time of keeping forms on monoliths and of keeping surfaces wet.

Test Cylinders.—As every minute detail of the processes of sampling, transporting, making, curing, and breaking cylinders affects the record of

breaking strength, the detailed account of the process, as described by Mr. Clifford F. Little, the Inspector who made all the cylinders at Auxiliary Lock 41 in 1929, is given in Appendix I. The standard cylinder, 6 in. in diameter and 12 in. high, was used for that and all other cylinders to which reference is made in this paper.

All the methods used were intended to be as prescribed by the Standard Specifications of the American Society for Testing Materials (Serial Designation, D 138-26, T), but, for obtaining uniformity, a detailed explanation or description of every movement was found desirable.

Due to defective bottoms, the entire first lot of moulds, purchased in 1925, was discarded. The second lot, purchased in 1926, from another maker, gave some trouble of the same kind, which is believed to account for some of the defective cylinder records from 1925 to 1928, inclusive.

A glimpse of the work and some further idea of its character may be obtained from Fig. 1. It shows the lower face of a part of the upper gate miter wall, a little of the finished face of the upper end of the right chamber wall, and the concrete forms erected for the head-house which includes the operating machinery of the upper filling valve. The masonry up stream from the gates is the wall of the old lock with its new concrete top which converted that lock into an upper guide-wall for the new lock. Fig. 2 shows the appearance of more of the face of the right chamber wall just after the forms and before the tie-rods were removed.

The specification for concrete in Auxiliary Lock 41 provided for about 24 cu. ft. of gravel instead of 25 cu. ft.; acting at his own suggestion and without change of specification or price, the contractor furnished double-ground cement instead of the standard required under the specifications. A comparison of the relative strengths of the 28-day cylinders and the cement with which they were mixed in 1929 (and some in 1928) and the relative strengths of the cements used in the corresponding years indicated clearly that the cement is usually the factor of first importance. Of course, it should always be realized that with good cement other errors may ruin the concrete.

In 1928, a particularly bad lot of cylinders was tested in a series made at one dam despite the constant vigilance maintained to prevent such errors. They were made and cured by a trained engineer, but due to other duties, he entrusted, to an experienced laborer, the task of transferring them from the seasoning bin to the place at which they were packed for shipment to the testing laboratory. Although the laborer had been around the laboratory and the work for months, he had failed to absorb the essential character, the purpose or importance of the cylinders, and after moving one to two dozen he was discovered with a pair in a sack giving them only a little better treatment than is customary with dull picks going to a shop.

Irregularities in the Strength of Concrete.—The principal causes of the great variations that occur in job concrete (as indicated by cylinder tests made on this work during four years) are the corresponding variations in:

(*a*) Strengths of cements of different brands, from different bins and from different parts of the same bin;

(*b*) Water content of aggregates, resulting in variations in water content of the batches;
(*c*) Correct measurement of aggregates (especially of sand), the water. and count of bags of cement for each batch (and the resulting water-cement-ratio);
(*d*) The grading of aggregates;
(*e*) The cleanness of the aggregates and the quality of the water;
(*f*) The time of mixing the concrete; and
(*g*) The sampling, fabrication, curing, curing temperature, handling, or breaking of the cylinders.

It is evident that these variations may be grouped into two principal classes: (1) The variations in the natural materials from which the cements are manufactured, in the natural grading of aggregates and content of silt and other foreign matter, and in the quality of the water as found in the river or other source; and (2), the variation in men, the competency and care exercised by the men performing all the many duties required in the entire process of preparing and assembling materials, transporting, mixing, and placing the concrete, and also by the opinions and attitude of those in supreme authority relating thereto. Lack of knowledge or failure to detect and regulate irregularities in one or all the foregoing seven conditions is responsible for the nearly universal failure of engineers in their effort to specify, manufacture, and place concrete as good as any specified quality.

When materials of reasonably uniform quality are provided, the writer believes that men can now be so well trained as to manufacture concrete (using laboratory methods) with the same certainty of producing a uniform product as a bread-maker can, for example; and on the job, with great irregularities in materials, such uniformity as is indicated in Table 1 should be obtained. After such uniformity is obtained and the lowest record in 50 or 100 cylinders is above the designed strength, then and only then should an economy be effected by reducing the proportion of cement or otherwise. When such reductions are contemplated, the inspector should be sure to maintain the designed standard.

Cement.—An account in some detail of the field methods used and results obtained in the construction and management in assembling materials may be useful for reference and comparison. All cement for the Ohio River improvement was purchased under specifications and contract, and was sampled and tested by professional laboratories. Samples were taken from every bin of cement and reports were made as to chemical tests, physical tests for fineness, time of setting, and tensile strength of 7 and 28-day briquettes.

Table 2 is an abstract of the cement test reports for 1928 and 1929 at various dams as noted.

Curves were plotted showing fineness and strength tests for 7 and 28-day briquettes. Due to the difficulty of storing and tracing shipments the bin number of the cement in each cylinder was not kept, except for the year 1929. Variation in fineness and in strength between the different kinds of the same cement as well as differences in brands, are more readily and clearly understood by plotting them as curves than by the tabulations of figures

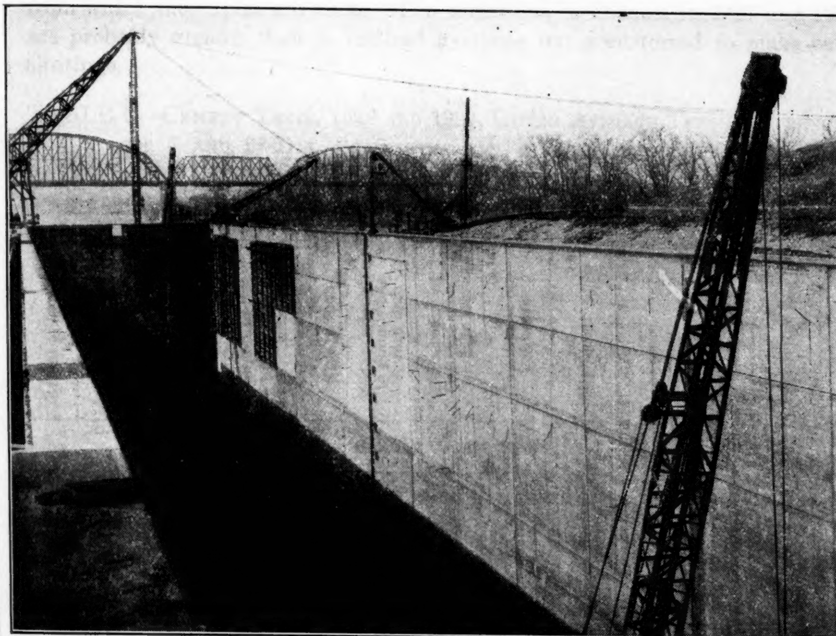

FIG. 1.—CONSTRUCTION VIEW OF AUXILIARY LOCK 41, LOUISVILLE, KY.

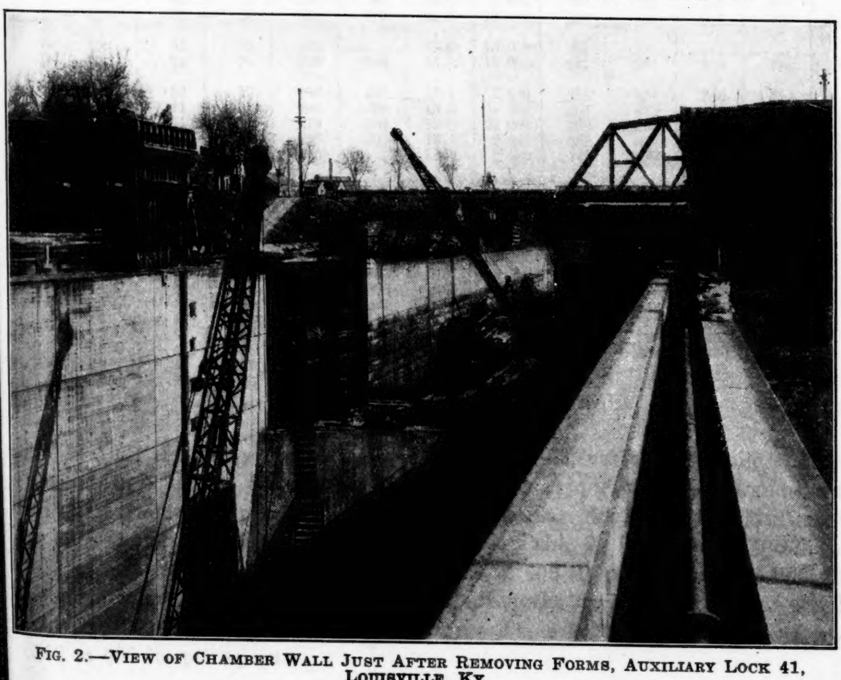

FIG. 2.—VIEW OF CHAMBER WALL JUST AFTER REMOVING FORMS, AUXILIARY LOCK 41, LOUISVILLE, KY.

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TABL

Brand of cement
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from which the curves are made. The variations in fineness in 1927 and 1928 are probably greater than is realized by those not accustomed to make such plottings.

TABLE 2.—CEMENT TESTS, 1928 AND 1929, GIVING AVERAGE TENSILE STRENGTH FOR 7 AND 28-DAY BRIQUETTES AND FINENESS MODULUS FOR EACH BIN OF EACH CEMENT USED

(Mix: 1 part of cement and 3 parts of Ottawa sand.)

Brand of cement	Bin No.	Dam	Number of tests	7-day strength, in pounds per square inch	28-day strength, in pounds per square inch	Percentage of residue on a 200-mesh sieve	Quantity in bin, in barrels	7-Day		28-Day		Average fineness
								Minimum	Maximum	Minimum	Maximum	
CEMENT USED IN 1928												
A	11	51	20	315	424	10.3	4 000	267	339	385	447	10.3
B	4	51	18	277	384	16.6	3 500	241	317	353	413	
B	12	51	15	261	370	15.5	6 000	238	279	351	388	
B	13	51	15	269	378	15.7	6 000	242	282	356	390	15.9
C	2	51	80	316	421	13.4	16 000	270	405	381	459	
C	6	50	45	322	378	11.9	9 000	297	354	336	404	
C	12	46, 51	60	314	421	12.6	12 000	280	339	388	448	
C	3	50	14	322	390	10.9	2 800	302	347	358	424	
C	4	50	14	291	339	12.0	2 800	272	320	326	367	
C	10	50	24	350	393	12.9	12 000	308	405	363	427	12.7
D	19	52	90	283	....	15.5	18 000	261	322	....	....	
D	20	52	75	278	389	16.8	15 000	244	320	309	424	16.1
E	18	53	12	287	395	13.1	2 400	251	305	366	412	
E	31	41	125	290	398	15.5	21 000	257	331	369	433	
E	34	41	125	279	384	19.0	21 000	250	399	360	423	
E	17	53	12	296	397	13.1	2 400	261	330	365	421	13.3
E	23	53	13	268	....	14.4	2 500	247	289	....	....	
CEMENT USED IN 1929												
F	4	41	16	369	465	3.20	9 000	340	403	420	500	
F	4 *B*	41	16	279	433	5.57	8 500	270	293	422	442	
F	4 *C*	41	16	266	399	9.55	8 500	253	275	388	408	
F	5 *C*	41	16	332	421	5.73	8 500	322	342	412	433	
F	6	41	34	326	461	5.84	17 500	303	360	443	483	
F	6 *B*	41	16	318	414	3.54	17 000	308	328	403	423	
F	10 *B*	41	16	323	413	3.77	8 500	313	333	430	447	
F	19	41	16	394	468	3.52	8 500	383	405	463	477	
F	6 *A*	41	20	299	420	6.25	20 000	292	312	407	432	5.46

In 1927 the 7-day briquettes of all brands used, varied in tensile strength from 241 to 345 lb. per sq. in.; and for 28-day briquettes of all brands used, the variation was from 326 to 455 lb. per sq. in. In 1928, the variation in tensile strength for 7-day briquettes was (see Table 2) from 238 to 405 lb. per sq. in. and for 28-day briquettes, from 309 to 459 lb. per sq. in. In 1929, for 7-day briquettes of Cement F only, the strengths varied from 253 to 405 lb. per sq. in., and for 28-day briquettes (see Table 2), the variation was from 388 to 500 lb. per sq. in.

In addition to the contract tests, complete field tests of cement that had been kept over winter were made according to the Ohio River General Specifications. Furthermore, two samples were taken from each car of cement as received at the work, one from each end of the car, and tested for soundness after the method specified[6] by the American Society for Testing Materials before being used. The use of cement stored over winter almost invariably resulted in low-strength concrete even when one or two additional bags of cement per cubic yard of concrete were used. This fact is painfully accentuated by the plottings in Fig. 3.

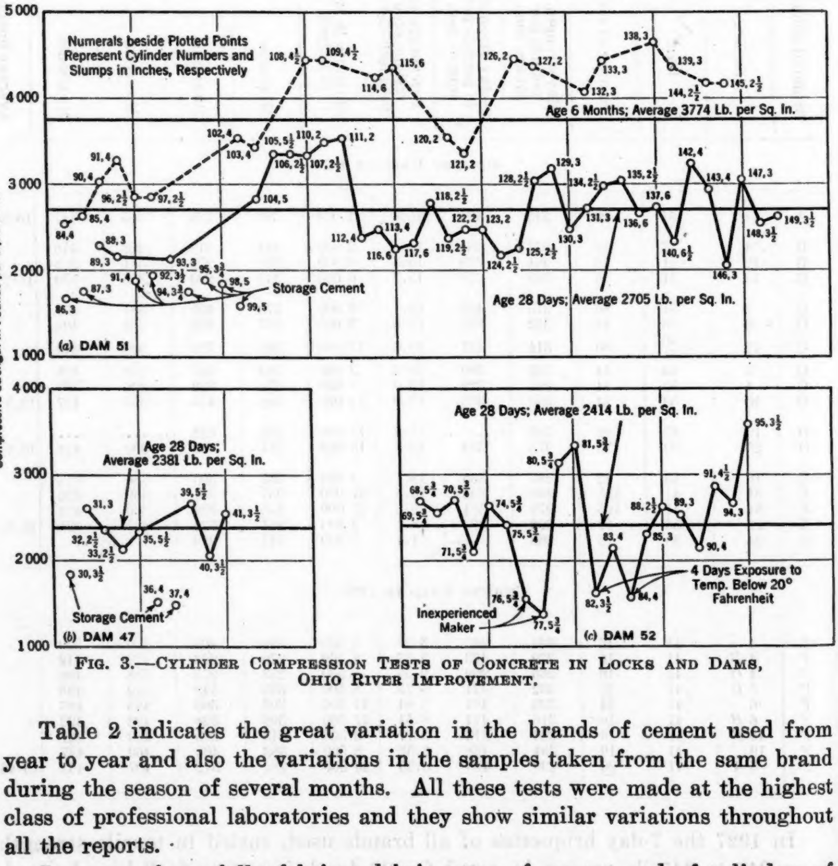

FIG. 3.—CYLINDER COMPRESSION TESTS OF CONCRETE IN LOCKS AND DAMS, OHIO RIVER IMPROVEMENT.

Table 2 indicates the great variation in the brands of cement used from year to year and also the variations in the samples taken from the same brand during the season of several months. All these tests were made at the highest class of professional laboratories and they show similar variations throughout all the reports.

In cases of carefully fabricated briquettes of cement and of cylinders of concrete made of cement from the same bin, although the two strengths do not uniformly correspond, it is not believed that all the variation is due to the cement. Frequently it is due to numerous causes, such as irregularities in water content, grading of aggregates, and other irregularities described elsewhere in this paper. However, the variation in cement probably causes the

[6] Standard Specifications, 1924.

most frequent and the greatest variations in the strength of the concrete where there is reasonably competent supervision. With incompetent or careless supervision the variation in water content is, no doubt, the condition which causes the greatest variation in strength and which is probably causing more concrete to fall below the proposed strength than all other conditions combined.

TABLE 3.—QUALITY OF FINE AND COARSE AGGREGATES

Description	Lock 41A	Dam 46	Dam 47	Dam 49	Dam 50	Dam 51	Dam 52	Dam 53
Average percentage of voids in dry loose gravel	36.4	34.8	31.3	36.5	35.8	39.3	40.8	39.1
Average percentage of voids in dry and rodded gravel	....	33.3	27.8	30.3	34.8	37.1	37.7	33.4
Percentage of bulking of sand: Maximum	....	50.0	32.4	44.2	32.0	36.7	29.0	41.4
Percentage of water used for maximum bulking	....	6.0	6.0	5.0	5.0	4.0	6.0	3.0
Percentage of bulking, inundated.	....	0.0	—2.7	0.0	2.0	6.3	0.0	0.0
Percentage of water used to inundate	....	20.6	25.6	29.4	27.7	44.0	31.7	36.1
Percentage of bulking of gravel	....	2.9	4.0	< 1.0	6.8	6.4	....	....
Percentage of water used for maximum bulking	....	4.0	4.0	......	4.0	4.0	....	....

TABLE 4.—SIEVE ANALYSES (PERCENTAGE RETAINED) AND AVERAGE FINENESS MODULUS OF SAND AND GRAVEL

Sieve	Dam 46		Dam 47		Dam 49		Dam 50	
	Sand	Gravel	Sand	Gravel	Sand	Gravel	Sand	Gravel
1½-in.	00.00	00.00	00.00	00.00	00.00	09.75	00.00	......
¾-in.	00.00	8.70	0.40	10.11	00.00	19.55	00.00	19.73
⅜-in.	00.00	79.77	5.05	54.90	00.00	68.35	1.39	88.74
4-mesh	00.00	100.00	8.50	89.47	3.10	95.60	2.34	99.84
8-mesh	17.70	100.00	24.80	95.15	13.75	98.05	7.81	100.00
14-mesh	27.70	100.00	39.27	96.92	24.90	98.90	15.55	100.00
28-mesh	40.50	100.00	54.50	98.03	44.30	99.15	48.20	100.00
50-mesh	95.60	100.00	90.55	99.47	95.10	99.45	96.17	100.00
100-mesh	100.00	100.00	98.62	99.90	99.75	99.75	100.00	100.00
Average fineness modulus	2.82	6.88	3.22	6.44	2.80	6.88	2.71	7.08

Sieve	Dam 51		Dam 52		Dam 53		Auxiliary Lock 41	
	Sand	Gravel	Sand	Gravel	Sand	Gravel	Sand	Gravel
1½-in.	00.00	00.00	......	7.07	......	3.15	....	7.1
¾-in.	00.00	10.48	......	52.01	......	48.29	....	46.4
⅜-in.	00.00	69.68	1.84	90.61	......	88.05	....	82.6
4-mesh	00.00	100.00	4.59	98.01	6.48	96.98	1.6	98.3
8-mesh	7.42	100.00	21.36	99.91	11.56	99.23	16.0	99.9
14-mesh	22.60	100.00	34.86	99.97	15.43	99.80	35.8	100.0
28-mesh	48.09	100.00	58.37	99.98	24.90	99.87	69.7	100.0
50-mesh	95.21	100.00	98.29	99.99	95.07	100.00	93.7	100.0
100-mesh	99.75	100.00	99.79	100.00	99.76	100.00	98.5	100.0
Average fineness modulus	2.76	6.80	3.15	7.47	2.53	7.30	3.15	7.34

Aggregates.—Table 3 gives the average of several samples of the aggregates (Ohio River sand and gravel) used at each dam with which the concrete, shown in Table 2, was made. Table 4 presents characteristic sieve analyses for the aggregate used at eight of the tests. The specifications stipulated that, "gravel or broken stone shall range in size such that at least 95 per cent. will pass through a 3-inch diameter ring, and not more than 15 per cent. will pass a No. 4 sieve".

When the aggregates are taken from the bars of the river, as was done on these works, their grading to a much greater refinement than about as indicated in this specification is not believed to be worth the cost in improved quality thereby obtained. However, there is no question that every barge load should be drained sufficiently to prevent change in the water-cement-ratio. When inundators were used for measuring sand the rule was for 6 hours or more of drainage of gravel to prevent excess water in the mix. Gravel long submerged in back-water was re-washed. Such gravel by test was found to reduce the strength greatly.

Tools for Placing.—When placing concrete it is most important to have a sufficient number of men in the form at all times, and that each man should be trained for his duties. They should know especially how to dump buckets and chutes, and how to spread concrete without causing it to segregate. Each spader assigned to a particular part of the face should be made responsible for that section and should be made to understand that the central mass must be thoroughly spaded and trampled to release entrapped air. The foreman should understand how to remove excess water and how to solve all other field problems affecting the quality and appearance of the finished work. There should be plenty of tools in good order for men on duty and some extras. The principal tools required on the Ohio River improvement were D-handle shovels, D-handle and long-handle spades, long wood spades, with blades about 4 by 10 in., and (for use in thin walls) bars and light hammers. A carpenter was stationed to work in the forms during the time that concrete was being placed.

Mixers.—The mixers were all of the "batch type machine" required to produce concrete of uniform consistency and workability. The mixing was continued for not less than 2 min. for a 2-yd. mixer, $1\frac{3}{4}$ min. for a $1\frac{1}{2}$-yd. mixer, and $1\frac{1}{2}$ min. for a 1-yd., or smaller, mixer, after completely charging and before beginning to dump. The batch was dumped into buckets on cars, lifted from the cars by derrick; and dumped (usually from bottom-door buckets), direct into place or into a hopper at the top of the form. From the hopper it was then conducted into place by a hose chute from 8 ft. to a maximum of about 25 ft. long, great precaution being required in all movements to reduce the segregation to the least possible. At the end of the specified mixing period for any batch, in case the dump bucket had not arrived and been made ready for receiving the charge, the mixing was continued until the bucket was ready, thereby improving the concrete, but introducing another factor for irregularity in strength.

The mixer for Lock 41 was of 2-yd. capacity, that for Lock and Dam 52 of 1¼-yd. capacity, and all the others were of 1 and 1½-yd. capacities.

Some instructions were given the foremen and men operating the mixers, but usually this was not begun early enough. The lack of trained and dependable men was a frequent cause of defect, which should be remedied by selecting the highest class of men procurable for experience and intelligence, and well known for integrity and reliability. Men who think the water-cement ratio useless, or that wet concrete is "good enough", should not be employed. They are especially objectionable in any position of authority. The crew on the concrete mixer and batch-placing forces in the forms should be selected with the greatest care as to experience and reliability some days in advance of commencing the work. Ordinarily, with competent inspectors, the direction of the principal men who are assigned to measure the materials, and the men in the forms, spreading, spading, and finishing the concrete, do most thorough work when under the direction of the inspector. The form inspector's proper position is close enough to the men to speak with ease to any one.

Throughout the season of 1928 Inspector R. W. McBeth had charge of the inspection of concrete materials, the mixing and placing of the concrete at Dam 51 and, during the season of 1929, he had similar duties at Auxiliary Lock 41. His recommendations for the proper inspection of concrete construction is described in Appendix II.

Conclusion

It is most fortunate for the artisan who can turn concrete mixing and the fabrication of cylinders into a game, or even more fortunate when he can, also, convert it into an art based upon highly scientific knowledge verified by laboratory experiments, as was done by Professor Abrams.

When the people who are paying for the concrete production of this country realize sufficiently the value of such skill, art, and knowledge as to demand uniform quality for soundness and a fixed minimum for strength, it is probable that the class of training indicated in this paper will be so perfected and extended as to make uniform products in concrete as usual as in other materials of construction.

The writer wishes to express here his appreciation of the great assistance of his associates with whom he has had much pleasure in accomplishing the tasks incidental to this class of work during many years.

APPENDIX I

Making and Seasoning Concrete Test Cylinders in the Field

By Clifford F. Little,[7] Esq.

The method given herein has been applied in moulding and seasoning test cylinders made from specimens of the concrete used in the construction of Auxiliary Lock 41. The process may be described in steps as follows:

[7] Insp., U. S. Engr. Office, Louisville, Ky.

(*1*).—*Collecting the Sample.*—After the first batch of concrete has been dumped the inspector hands two pails to one of the laborers in the form, giving him directions to fill them. The sample is promptly carried to the laboratory.

(*2*).—*Making a Slump Test.*—Upon arriving at the laboratory it is dumped from the pails to the dampened concrete floor and then immediately turned over by eight strokes of a large trowel. This operation will rectify any segregation which may have occurred in transportation and by dumping the sample from the bucket. Immediately thereafter, its slump is tested by filling the slump cone with three, 4-in. layers, puddling each layer with twenty-five strokes of a bullet-pointed ⅝-in. rod of standard design. After the last layer has been puddled, the surplus concrete is struck from the top of the cone with the rod. Then, by use of the handles, the cone is slowly raised vertically until it clears the top of the concrete, and is placed beside the specimen. The rod is laid horizontally across the top of the cone, and, in that position, its distance above the top of the specimen is recorded, in inches, as the slump of the concrete.

(*3*).—*Preparing the Cylinder Moulds.*—The moulds are prepared for concrete before the sample is brought from the form. Every particle of old concrete is removed and then the inner surface is rubbed with a light oil. This treatment reduces the tendency of the new concrete to stick to the mould. Bottom plates of the brass and of the heavy cast-iron moulds were found unsatisfactory on the Ohio River work and they were replaced by steel plates 8 in. square by ½ in. thick, machined to a true plane on one side. These produced very satisfactory results in securing true ends to the cylinders so that they rarely required re-capping.

(*4*).—*Making the Test Cylinder.*—Care should be exercised when placing the mould on the bottom plate to see that no small particles of sand have been caught between the bearing surfaces of the plate and the mould. After making these precautions, spread the concrete which was used to make the slump over the remaining concrete of the sample. Proceed immediately to make the cylinder by placing concrete from this sample in the mould in three, 4-in. layers, puddling each layer with twenty-five strokes of the ⅝-in. rod. Care should be exercised to remove all pieces of aggregate that are more than 1 in. in diameter. After puddling the last layer, press down the top concrete in the mould with the blade of the large trowel thus removing all excess concrete and any air-pockets left by the rod. When all the excess concrete has been struck off with the trowel, give the top end a final finish by rubbing the rod over it.

(*5*).—*Capping and Storing the Cylinder.*—After the cylinder is made, prepare a stiff, neat paste for capping, with cement from the same bin as that used in making the cylinders. Soon after the cylinders and paste have shown the final set (by needle), the paste is re-worked until it is soft, and then it is spread on top of the cylinders in a thin layer, about $\frac{1}{16}$ in. higher at the center than on the edges. A sheet of thin oiled paper is spread over the top and one of the steel plates is placed over this paper. The plate

is then pressed down and slightly turned from one side to the other until it has a firm bearing on the entire edge of the mould.

Thereafter, the cylinder is not disturbed for 24 hours. At the end of this period remove the moulds carefully, wipe all loose particles of sand from the cylinder, and place it on end on a perfectly plane surface. Place one hand on top of the cylinder and rock it slightly in all directions. Should the end be imperfect, a tilting motion of the cylinder will be felt, indicating that the end is not a perfect plane. It may be made perfect by rubbing it over the plane surface of the steel plate until no rocking motion can be detected by observation or by "feel".

The ends should also be tested to see that they are perfectly parallel. This may be done with a carpenter's square or by placing the cylinder in the testing machine. If they are not perfectly parallel, they should be made so by rubbing or by re-capping. After both ends of the cylinder have been treated in this manner, so that they will coincide perfectly with the plane surface of the testing-machine bearing-plates, they are marked and stored in damp sand which is kept within 15° of the average temperature of 75° Fahr. They are not again disturbed until the shipping date, when they are re-packed in shipping boxes filled with sand and shipped by truck to the testing laboratory. There, again, they are transferred into sand storage of about the same temperature as that in the field laboratory until they are broken in the testing machine.

APPENDIX II

Standards of Inspection for Concrete Construction

By R. W. McBeth,[8] Esq.

Hostility is one of the greatest obstacles to the control and inspection of concrete construction. Frequently, construction heads refuse to be informed, or to attend school. Indifference and hostility prevent the spreading of proper information and the co-ordination of operations between these workmen and the inspectors.

Plant Location.—The plant should be so installed as to get maximum output and this requires that every detail must be studied, and the installation made with care. Mixer frames must be secured firmly to solid foundations, and made plumb and level. Bearings and clutches, especially the tilting gear-clutches, need the closest attention. The charging hoppers should be set as close to the drums as clearance will allow. The throat of the hopper should be enlarged to the limits that the feed-opening of the drum will permit, and the tilting transfer scoop should be correspondingly enlarged, to prevent choking and spilling.

Mixing Water.—Water tanks should be cylindrical and set vertically, as nearly as possible over the drum feed, and the discharge pipe should be carried

[8] Insp., U. S. Engr. Office, Paducah, Ky.

directly from the tank to the top of the drum feed-opening, without abrupt angles or bends. It should end in a flattened nozzle situated so as to deliver the mixing water on the rear third of the drum. This will give a quick, clean delivery of water. The tanks should be calibrated and fitted with gauges and floats automatically reading to gallons. The gauge marker should work accurately and smoothly.

Batch Mixing.—The inundator operator should be carefully trained by one familiar with its mechanism and "kinks" of operation. The use of a separate cement charging hopper is of much value because it eliminates hopper fouling. A common fault of concrete plants is a slow rate of feed to the drum. Many charging hoppers require 40 sec., whereas it should be done in less than 25 sec. A batchmeter should be installed on the mixer, so that the inspector will have a chance to watch the mix. Stairs and ladders should be placed so as to afford quick and easy access to all parts of the plant. The interior of the mixer drum should be brightly illuminated, day and night. When the hopper gate is open a spotlight may be thrown diagonally down the hopper into the drum. In non-tilt mixers, this is the only practical method. If bag cement is used the best method of charging is to empty bags into a skip that dumps at the top of the hopper, clearing the stream of mixing water. The skip should be as long as possible to minimize the splashing of cement. A locking and recording batchmeter should be a part of the plant equipment.

There are many things to be done beside checking tally and ringing bells, and the tallying is certain to be inaccurate when supervision and operation constantly interfere with timing and checking. The water tank should be covered and the valves and piping should be kept in good order. The platforms for observing changes in the mix and consistency and for workmen should be well lighted. When mixing is suspended for an hour or more, the mixer drum, and all the platforms, the hoppers, or chutes, at the mixer and in the form, should be thoroughly cleaned.

Many samples of sand and coarse aggregate should be carefully selected so as to represent the extremes of materials proposed for use. When separated into their respective sieve sizes they exhibit the grading. Weights of both materials per cubic foot in "dry rodded", "dry loose", and "damp loose" states, must be ascertained; and, also, the bulking of the sand with the probable moisture range. Piled in barges, stock piles, and plant bins, the aggregate naturally separates into coarse and fine particles and it is useful to know the effect of the different sizes of materials in a mix.

In general, a batch of coarse material, following a run of fine, is likely to produce a coarse, wet mix, which can be placed with comparative ease, but which results in poor construction. Fine material, on the other hand, will produce a wet mix with wet gravel, or a dry mix if the gravel is dry or low in moisture content. This is due to the much greater surface area, which takes up water from the mix if it is low in moisture, but adds water if it is wet. In either case, a run of fine gravel in a batch will cause a

harsh mix, difficult to place, but one that produces good concrete if well compacted in place. However, it is usually difficult to get this tamping done properly.

To obtain good work, special care is required when the grading of sand or gravel is frequently changing. The specified measure of sand (equal to voids in gravel plus 15% of voids) will ordinarily give only a sufficient amount when a workable mass is well mixed; and, therefore, care must be used in making allowances for bulking. Sand thus proportioned should cover the maximum of voids likely to occur. Weighing batchers are preferable for obtaining accuracy in measuring coarse aggregate.

The mix should be watched and if the concrete is harsh, more sand is desirable. A slight reduction could be made in the gravel charge, but not quite enough to balance the increase of sand. Ordinary changes of ½ cu. ft. for fine or coarse aggregate will not unbalance the yield. Due care should be taken to see that the gravel "batcher" is set accurately. Variations in this unit will change the relative proportions twice as fast as the sand batcher, for a desired change of proportions in the charge.

A good method of determining proportions to be used at the beginning is the "trial method" devised by F. R. McMillan, M. Am. Soc. C. E., and recommended as standard by the Portland Cement Association. By this method, the mix is made in small batches to determine the correct point of workability with several water ratios. Coarse and fine gravel is analyzed in the various proportions and water-cement ratios. Various mixes are studied until the inspector is satisfied, and then the quantities required are determined for the batches so as to duplicate the best trial mix, in the actual structure.

Inspection During Operations.—When the plant has started, the slump test should be made of each batch until they are satisfactory. The batch should be observed carefully at the mixer, in the bucket, and when it is being placed in the form. The inspector should note how it acts, and should not hesitate to ask the foreman and form inspector, or the placing gang, how it behaves. He should spade and tramp it himself. In making the slump on trial a well-balanced mix should not slump off on the side and show gravel, provided the cone is lifted smoothly and steadily. Such a tendency indicates insufficient mixing or insufficient sand. By remembering that 4.7 lb. of cement is 0.01 of a 5-sack batch (1 cu. yd. as specified) and knowing the weight of all other materials per cubic foot in all conditions, computations required for changes are made easier. The material bins should have slopes of not less than 50° to the batcher gates. The grading of aggregates may be improved or injured by methods of loading and unloading.

The location of material supply piles and plant machinery is very important to efficient operation. For a river coffer-dam both should be situated so as to protect supply barges from the current, allowing easy handling for turning and cleaning off the barge. The relative position of barges, derrick, and mixer must be such that a minimum of movement is required to place material in the bins. By care in this operation the grading of gravel may be improved,

or, by lack of care, it will be injured. To have a good signal man who understands this process of improving grading, is very advantageous. The bin should be in view of the derrick operator, and his operating house should be set so that he has a clear view of the plant and barge.

Concrete forms should be wire-brushed and oiled every time they are stripped. They should be kept wet so that cracks do not open up and the panels warp. This may cause complaint, but it is worth while, especially in close forms, and where finish is desired. Where reinforcing steel is used, the forms must be oiled before the steel is placed. Mortar should be placed in the bottoms of all lifts for bond; it is difficult to obtain neat cement, when so used, in sufficient quantity. Mortar will help correct any effect of dry batches when starting a monolith. Lack of control at the mixer may result in occasional dry or wet batches in the form, especially at the start, should the moisture content of the aggregates change quickly.

Plant Control Equipment.—The inspector or control man should have a laboratory on the charging floor, with a high-grade laboratory scale of large capacity. It should have a triple beam, graduated in decimals, a 5 000-gramme torsion balance measuring to ½ gramme, with special platform and a one-point suspension unit for specific gravity determinations. A 6-in. cylinder with a capacity of 0.2 cu. ft., fitted with double weirs and "strike off" cover, is useful. The following should also be on the list: a 1-cu. ft., bored steel measure; a 0.1-cu. ft. measure with scoops and rods; two slump cones; scale and balance scoop pans with counterweights; buckets and plenty of pans, large and small; an oven taking the largest pans, if practicable, with an electric heating unit of 1 800 watts and with three or four-point control; two 50-cu. cm., and three 150-cu. cm., cylindrical graduates; a pycnometer; a dozen mason jars; a dozen 8-oz. graduated prescription bottles, with corks; and a 1-gal. jug, with a cork. A full set of standard sieve analysis screens, with pan and cover, 12 in. in diameter; two steel shaker-frames, one for sand and one for coarse aggregate sieve nests, with electric vibrator for shaking samples of aggregates, are essential. Brooms and heavy dust brushes, a sink and large floor pan housed in a well-lighted and practically dust-proof room on the platform—all of these items comprise the essential equipment needed for plant control.

If thorough preliminary examinations have been made, the control man will have most of the necessary reference data and charts at hand; otherwise, he will have to develop them in the course of his control work. If the scales and balances are as specified herein, no conversion tables will be needed, and the work will be simplified and expedited. At any time the control man can catch a sample of either aggregate from the batch as it falls, and from it, in a few minutes, he can have any data desired.

All the apparatus mentioned is designed for rapid and accurate work, and, with some practice, two or more determinations can be made almost simultaneously, enabling the inspector to keep close check on all characteristics of the materials in use, and quickly to detect any changes.

With weight-batching, the proportions can be changed completely between batches if necessary, to meet any material changes in grading. The control man should be chosen not only for his knowledge, but for a deep interest in the work, and a known desire to produce the best possible concrete at the lowest possible cost. Such a man will have an enjoyable job. He should be in charge of the plant, including the batchmeter and the repair work. With such equipment, the inspector may be able to obtain satisfactory results unless by chance some one has omitted from the contract specifications one or two essential paragraphs.

The control of concrete is a great game and when one can look at thousands of yards of good concrete in a big structure, with few defects, he will feel that it was well worth the effort.

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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

REPORTS

SPECIFICATIONS FOR BOUNDARY SURVEYS[1]

PREPARED BY
COMMITTEE OF SURVEYING AND MAPPING DIVISION ON BOUNDARY SURVEYS

Boundary surveys may relate to real property boundaries, political boundaries, or riparian boundaries. These specifications refer particularly to real property boundaries. In time, they may be found helpful when applied to political and to riparian boundaries.

In order that the public may be protected, there should be a common understanding among all persons interested in boundary surveys as to the standards to be observed and the procedure to be followed. This understanding should deal with fundamentals in a brief and simple manner. When it has met with the approval of competent and unselfish practitioners, it should be widely circulated among those interested in real property. It should be regarded as something in the nature of a moral constitution to be respected and supported by all good engineers and surveyors.

Those who are responsible for the prosecution of boundary surveys understand that they are faced by handicaps. The surveyor suffers because of a lack of National, State, and local organization. Important decisions of the Court relating to real property boundaries become matters of public record and a part of the law of the land. The advice given by surveyors (who, as arbitrators, settle many controversies relating to such boundaries) is lost to the profession and to the public because it is seldom reduced to writing and entered on the public records.

Public opinion supporting accuracy, substantial monuments, control, and complete records of surveys, may be developed by a campaign of education. Those engaged in professional work of this kind should combine in the organization and support of such a campaign. Their efforts will be rewarded in so far as they may convince those who provide the money for boundary surveys that great economies are involved.

The surveyor who makes a boundary survey of real property depends upon monuments to guide the progress of field work. These monuments may represent local control, or they may mark the boundaries of real property owned or controlled by his clients or by neighboring freeholders. Sections of any boundary between adjacent monuments may be straight lines or curves.

[1] These Specifications are published for the information of the membership, and others, and discussion thereon is invited.

They run consecutively from monument to monument, finally returning to the point of beginning, thus completing a closed figure.

Corrections for Errors

The most valuable check to be applied to a boundary survey is found in the relative magnitude of the computed closing errors; in general, the better the work the smaller the closures. The resulting discrepancies are due to errors in linear and angular measurements. Corrections are made to the measured angles and the measured lengths of the sides for the purpose of eliminating the closing errors and thus making the notes apply to a closed figure. The procedure is supported by the mathematical theory of probability. It appeals to the logic of the average mind because all who are able to visualize an area of land realize that its boundaries should form a closed figure. Even if only one boundary line of a parcel of real property may be questioned, one or more closed surveys should be made before advice is given or evidence offered.

Errors are generally expressed as ratios or fractions. If an error of 1 ft. occurs in measuring a line 1 000 ft. long, the error is said to be 1 in 1 000. No field work relating to boundaries should be considered as a survey when the accuracy is less than 1 in 1 000. A final survey which locates monuments and supports a description of real property boundaries should maintain an accuracy of 1 in 5 000, or better. An accuracy of 1 in 10 000 is recommended since it may be uniformly maintained when modern equipment is used by qualified surveyors. First-order triangulation, under the specifications of the United States Coast and Geodetic Survey, maintains an accuracy of 1 in 300 000, or better, for base-line measurements, or 1 in 25 000 for the lengths of the triangle sides. Where such control is available, local practitioners need have no doubt as to the reliability of monuments established thereby. One of their great responsibilities is to extend local control so that it will be of greater service to the public and a matter of lasting credit to those performing the work.

Standards of Accuracy

The main network of local triangulation systems which connects with the monuments established by the Federal Government should be governed by specifications nearly as exacting as those applied to first-order triangulation by the United States Coast and Geodetic Survey. These may involve quadrilaterals having sides approximately as long as those common to the systems extended under Federal authority.

As detailed triangulation control is provided for smaller areas the error ratio may be greater. If the sides of the quadrilaterals are approximately 5 or 10 miles in length, the ratio may be 1 in 150 000 for base-line measurements, or 1 in 10 000 for the lengths of the triangle sides. In smaller systems where quadrilaterals are from 1 mile to 2 miles in length, the ratio may be increased to 1 in 75 000 for base-line measurements, or 1 in 5 000 for the lengths of the triangle sides. Triangulation systems extended to furnish

control for boundary surveys for real property should be kept within these limits, that is, between 1 in 300 000, or better, and 1 in 75 000, or better, for base-line measurements, with the corresponding ratios for lengths of triangle sides.

The dimensions of limiting errors applied to boundary surveys will be influenced by the accuracy of the triangulation control and by the demands of freeholders and others interested in real property. It would not seem logical, in general, to prescribe smaller limiting errors for boundary surveys than those applied in the triangulation control. However, if the surveyor is directed to determine the locations of monuments to mark the boundary of real property within narrowly prescribed limits, he may find it necessary to use specifications more exacting than those followed in connection with the extension of local control.

Rectangular Co-Ordinates

The Federal system of first-order triangulation has been extended along the Atlantic and Pacific Coasts and along numerous parallels of latitude and meridians of longitude. When it is completed, within the next twenty or twenty-five years, it will connect many points distributed through the country as they are spaced on the curved surface of the earth. Several progressive cities have indicated how this system of Federal control is to serve groups of freeholders who wish to take advantage thereof. These cities have adopted rectangular co-ordinates to which local control and monuments marking real property boundaries are referred.

The areas served by a single system of rectangular co-ordinates may run from 3 or 4 to 40 or 50 sq. miles. The shape and size of these areas will depend upon the judgments and desires of local people. The origins or centers of co-ordinates of these systems are to be connected by triangulation to the Federal system of control. A third system of triangulation is then spread over each area referred to rectangular co-ordinates. All monuments established in connection with the third system of triangulation and those set by local surveyors to mark the boundaries of real property are to bear the co-ordinates of the points they represent. Beginning with Federal control, the connecting systems will ultimately have complete development, as follows:

1.—Federal system of triangulation.
2.—Triangulation system connecting the Federal system and origins or centers of systems of co-ordinates.
3.—Triangulation control for the areas referred to systems of rectangular co-ordinates.
4.—Boundary surveys based on rectangular co-ordinates within each such area.

Responsibility of the Surveyor

When those engaged in local surveys are unable to secure the necessary funds to provide triangulation control, they should make the best of an unfortunate situation. They should agree among themselves as to the location and character of monuments to which all local surveys may be connected. These monuments will serve them until triangulation control is provided. If

they are carefully located they may be accepted as triangulation stations, or, at least, connected with such stations when a system of control appears.

There is no conflict between the professional responsibilities of surveyors and attorneys-at-law. The lawyer is concerned in the settlement of controversies arising from claims to real property. The surveyor obtains scientific data in the field, applies the necessary tests for accuracy, completes computations, and prepares such maps and charts as may be helpful to his clients.

Surveyors should always prepare the descriptions of the boundaries of real property. When others assume to render this service to freeholders, they should be required to indicate to local surveyors the points on the ground where monuments are to be set to locate the boundaries concerned. This work should be performed in the presence of all interested freeholders and the surveyors employed by them. Surveyors should not accept descriptions of boundaries of real property, which do not clearly define the character and position of monuments. Monuments fix real property boundaries on the ground. When the positions of monuments have been determined, descriptions of boundaries can be made.

Whenever an agreement is reached by all interested freeholders, monuments should be set and connected with local control. Complete field notes, computations, maps, and agreements relating to monuments set and described, should be prepared in documentary form, signed by all interested freeholders and the surveyors employed by them, and they should be recorded in a local office of public record.

Prospective purchasers of real property should insist that they be shown the monuments limiting the boundaries thereof while negotiations relating to the terms of the sale are in progress.

A manual, based on these specifications, including such modifications as are shown to be desirable in the ensuing discussion, is to be prepared for the convenience and guidance of surveyors.

Committee on Boundary Surveys,

C. T. Johnston, *Chairman*,

C. O. Carey,

H. Bouchard,

H. J. McFarlan.

March 23, 1931.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

EFFECT OF TURBULENCE ON THE REGISTRATION OF CURRENT METERS

Discussion

By Messrs. Robert E. Horton, Richard Van Vliet, and David L. Yarnell and Floyd A. Nagler

Robert E. Horton,[3] M. Am. Soc. C. E. (by letter).[3a]—The paper under discussion presents the results of three different lines of investigation of the relation of the velocity registered by current meters of different types to the velocity of the stream. It suffers somewhat from seeming, if not actual, contradictions of statement and from lack of clarity. It is also difficult to check some of the figures and conclusions owing to absence of the original tabular data of the experiments.

To avoid uncertainty or confusion it has been found necessary at times in this discussion to set forth briefly what the writer understands to be the meaning of some of the statements and results presented by the authors.

Current meters for measuring the velocity of water are of two general types. In the cup type, illustrated by the Price meter, the axis of the rotor (as distinguished from the axis of the meter), is normal to the line of flow. Such meters are actuated by the excess of dynamic pressure on a flat surface over that on a curved or conical surface having an equal projected area. In the screw or helical type, the axis is parallel to the line of flow. Such a meter is actuated by impact of water on the inclined surfaces of the vanes.

Current meters serve, among many others, for three important uses:

1.—Determination of points on stage-discharge relation curves or rating curves for open-section stream-gauging stations. For this purpose cable suspensions are generally used.

2.—Measurement of flow in artificial channels, such as head-races and tail-races in power plant tests. Such channels often contain artificial turbulence, which is sometimes violent.

Note.—The paper by David L. Yarnell and Floyd A. Nagler, Members, Am. Soc. C. E., was published in December, 1929, *Proceedings*. Discussion of the paper has appeared in *Proceedings*, as follows: February, 1930, by Charles S. Bennett, M. Am. Soc. C. E.; May, 1930, by Messrs. Brehon B. Somervell, N. C. Grover, Burke L. Bigwood, G. H. Nettleton, and C. Linden; August, 1930, by H. R. Leach, M. Am. Soc. C. E.; September, 1930, by Dr.-Ing. Ludwig A. Ott; November, 1930, by B. F. Groat, M. Am. Soc. C. E.; and March, 1931, by Forrest Nagler, M. Am. Soc. C. E.

[3] Cons. Hydr. Engr., Voorheesville, N. Y.

[3a] Received by the Secretary, February 27, 1931.

3.—Measurement of flow in channels, such as irrigation ditches, etc., commonly with smooth flow and slight turbulence. For these conditions rod mountings are frequently used. In the experiments described by the authors, rod mountings alone were utilized and, as was pointed out in the paper, the results and conclusions cannot with certainty be applied, unmodified, to meters used with cable suspension.

The most important use is in the measurement of natural streams. For this purpose the current-meter method is often the only one economically available. It permits the measurement of flow without loss of head and without disturbing natural conditions. Turbulence is nearly always present, particularly at higher stages. It seems worth while, therefore, to give some attention to the question of turbulence in natural streams.

The Nature of Turbulence.—In this discussion turbulence will be considered as consisting of eddies. For this purpose, an eddy may be defined broadly as a rotating mass of water regardless of the distribution of velocity within it, provided all the particles rotate around a common axis. A vortex is an eddy with a special velocity distribution such that for all radii, R, of rotating filaments the product, Rc, is constant, in which, c, denotes the tangential velocity of a filament at the radius, R. A vortex represents a stable system of motion in the sense that it can be destroyed only by friction. A part of the fluid may also rotate as a solid, that is, with uniform angular velocity at all radii. As the result of friction there may be a combination of a vortex and rotation *en masse* as, for example, a condition precisely intermediate.

Table 10 indicates the characteristics of three types of rotating fluid masses.

TABLE 10.—CHARACTERISTICS OF ROTATING FLUID MASSES

Type No.	Name	Relation of c to R	Relation of angular velocity, ω, to R	Relation of speed of rotation, N, to R
1	Free vortex......	$c = \frac{\text{constant}}{R}$	$\omega \propto \frac{1}{R^2}$	$N = \frac{c}{2\pi R^2}$
2	Rotation *en masse*......	$c = 2\pi N R$	$\omega = \text{constant}$	$N = \text{constant}$
3	Intermediate condition.	$c = \text{constant}$	$\omega \propto \frac{1}{R}$	$N \propto \frac{1}{R}$

The origin of eddies, particularly vortices in fluids, has been described by Dr. L. Prandtl, taking into account recent researches.[4] This need not be repeated further than to note that eddies originate where there is an obstruction to flow, such as a projection from the boundary of a natural channel, sufficient to produce a surface of discontinuity in the adjacent fluid. If there is a sufficiently marked difference in direction or magnitude of velocity on the two sides of this surface, an eddy is formed which grows in magnitude

[4] "The Physics of Solids and Fluids—with Recent Developments," by P. P. Ewald, Th. Pöschl, and L. Prandtl, authorized translation by Dougall and Deans, pub. by Blackie & Sons, Ltd., Lond. and Glasgow, 1930.

until it breaks away, carrying a mass of fluid with it. A new surface of discontinuity is then formed down stream from the projection and the operation is repeated.

The application of these results to the circulation of water in a natural stream will next be considered.

In a natural stream, eddies originate (except as hereafter noted) wholly around the margin of the cross-section and tend to travel normal to the boundary of the channel at the point of origin of the eddy, or at right angles to the line of flow. The normal component of eddy motion, is, however, combined with the velocity of the stream so that the resultant direction of motion is diagonally down stream and toward the center or surface of the stream. As the eddy travels, its energy is dissipated as heat. In a very wide channel, eddies thrown off horizontally at the sides may be dissipated before reaching the center of the channel. Eddies thrown off at the bottom, if weak, or if the channel is very deep, may not reach the surface. Strong eddies reaching the surface are spread out as visible "boils".

A vortex (like a smoke ring) carries with it part of the fluid out of which it is originally formed. Since eddies originate around the perimeter of the cross-section and hence in regions of low velocity and low unit volume energy, their propagation through the cross-section tends to equalize the velocity, as is in fact observed.[5] This theory of the circulation of water in rivers as influenced by turbulence has other important implications not relevant to the present discussion. One, however, may inquire: What becomes of the kinetic energy released by the equalization of velocities resulting from eddy cross-currents? On the other hand, the retardation of velocity due to viscous shear tends to equalize the velocities in a vortex and reduce that vortex to a simple case of rotation *en masse,* and any intermediate condition between that of a free vortex and that of simple rotation *en masse* may obviously occur. Changes taking place in eddy turbulence at different distances from the boundary where the eddies originate, have been studied experimentally.[6]

Mutual interference and coalescence of eddies do not greatly change the situation. They tend in many cases to accelerate the dissipation of energy and the disappearance of eddies. In other cases they accentuate the disturbance and may produce abrupt and violent changes or reversals of velocity such as are sometimes observed in current-meter work in violently turbulent water. Another interesting and important fact which, however, the writer has not seen described in engineering literature, is the "calving" of eddies of the vortex type. Energy changes due to external friction may cause the breaking up of an original vortex eddy formed at the boundary of the channel after it has risen part way or entirely to the surface. Under suitable conditions the original vortex eddy disappears, but it is succeeded by a series of small, active vortices, each containing a portion of the original vortex fluid. This phenomenon of "calving" of vortices may be successfully demonstrated

[5] "The Physics of Solids and Fluids—with Recent Developments," by P. P. Ewald, Th. Pöschl, and L. Prandtl, 1930, p. 281.

[6] "Caratteri della Turbolenza nelle Condotte Cilindriche," *L'Elettrotecnica,* No. 15, 1930, pp. 730–733.

by a simple experiment if carefully performed.[7] A minute drop of ink on the end of a toothpick is gently shaken into a glass of perfectly still water. A black vortex ring travels downward, expanding and presently breaking up into a group (usually five or six) of small, perfect vortex rings, radiating outward and downward. These, in turn, break up into other groups, the entire series resembling successive star clusters from a rocket. The breaking up of original eddies, therefore, is not followed necessarily by a heterogeneous or hit-and-miss system of cross-currents, but, in part, by a new order of active eddies. The writer believes that a theory of the behavior of current meters in turbulent flow predicated on the composition of rotary and longitudinal motion (assuming the rotary motion to be of the character described), will provide a reasonably accurate "model".

Effect of Artificial Turbulence.—The first part of the paper gives the results of a series of experiments wherein artificial turbulence was induced in the experimental canal by horizontal and by vertical paddles (see Fig. 11). In these experiments the mean velocity in the canal was 2.0 ft. per sec. Different types of meters were used, and with each meter a velocity determination was made at each of the eight points in the cross-section as indicated on Fig. 10. Tests were made first with each meter, using horizontal paddles, and these were duplicated. Tests otherwise identical with the preceding were then made with vertical paddles, which were also duplicated. The resulting indicated velocities at the index points are shown in Table 1. The means of these velocities are given in Table 11.

TABLE 11.—MEANS OF VELOCITIES AT EIGHT POINTS OF MEASUREMENT IN FLUME WITH PADDLES

(Units are in feet per second)

Meter (1)	Horizontal paddles (Test 8) (2)	Vertical paddles (Test 9) (3)
Small Price, electric	2.18	2.12
" " acoustic	2.36	2.28
Hoff	1.74	1.90
Small Ott	1.76	1.95
Medium Ott	1.63	1.75
Large Ott	1.77	1.73
Mean	1.91	1.96

The figures in Columns (2) and (3), Table 11, for a given type of paddle, show the results obtained by taking one-half the sum of the means of the velocities measured at the eight index points in each of the two tests.

It was generally found that the agreement between mean velocities at a given index point in the two tests with a given meter and a given type of paddle was excellent. Therefore, only the average of the two tests is included in Table 11.

[7] "The Splash of a Drop," by A. M. Worthington, Smithsonian Inst., Annual Rept., 1894.

Comparing the velocities obtained, respectively, with horizontal and with vertical paddles, it will be seen that the differences are small but significant in the case of most of the meters, but that they greatly exceed the allowable limits for accurate measurement in some cases, particularly for the Hoff and small Ott meters. If the average of the six determinations of the mean velocity in the channel for the six different meters is taken, the result with horizontal paddles is 1.91 ft. per sec., while with vertical paddles it is 1.96 ft. per sec. In view of the close agreement of these figures and their small departure from the true mean velocity of 2 ft. per sec., the writer believes that the differences in velocity shown by individual meters in this test with the two types of paddles are probably due mainly to fixed differences in distribution of velocity rather than to variations in the total amount or character of turbulence in the channel resulting from the types of paddles used.

There is evidence in Table 1 of a decidedly abnormal distribution of velocities in the channel. Furthermore, it appears probable that the turbulence produced by the paddles differed from that existing in a natural stream channel so materially as to make the applicability of these results in practice somewhat doubtful. This is unfortunate since these are the only experiments included in the paper in which turbulence occurs to any important extent in the water in which the meters were suspended.

Two conclusions, however, may be drawn from these experiments:

1.—In very turbulent water the small Price meter may over-register from 6 to 18%, while the Hoff and Ott meters may under-register by amounts varying with the type of meter from 2½ to 18 per cent. By methods somewhat similar to those used by the authors, Kirschmer and Esteren have confirmed[8] the over-registration of the Price meter and the under-registration of the Ott meter.

The Price meter over-registered more with horizontal than with vertical paddles, while the Hoff and Ott meters generally under-registered. This probably resulted from the difference of behavior of these meters in fixed cross-currents, as revealed by the third series of experiments.

2.—A second conclusion is that by the use of two or more meters so selected that over-registration of one kind is balanced by under-registration of the other kind, comparatively accurate results can be obtained with almost any degree of disturbance of the current by baffles or other obstructions. The use of two meters, one of which is known to over-register and the other to under-register, is a practice which has been followed by the writer and others in measurements of very turbulent flow in power plant tests and other similar situations. The authors' experiment so far as it goes, confirms the utility of this practice.

Energy Consumed in Friction as a Measure of Turbulence.—Correction factors to be applied to different types of current meters in use in turbulent water are greatly to be desired. In fact, the determination of such factors seems to be the proper primary object of experimental investigations such as those by the authors. It is to be regretted that they did not go much farther

[8] *Zeitschrift* des Vereines Deutscher Ingenieure, v. 74, pp. 1499–1504; also, *Engineering Abstracts*, Inst. C. E., Lond., No. 46, January, 1931, pp. 20–21.

along this line. It might be inferred that since there is no quantitative method of measuring turbulence, such correction factors, even if available, would be of little practical utility. Although the point seems to have been overlooked heretofore, the writer believes that an index of the amount of turbulence in a stream can readily be obtained from the ordinary slope formula. Take, for example, the Manning formula,

$$v = \frac{1.486}{n} R^{\frac{2}{3}} \sqrt{s} \qquad (12)$$

in which, v is the mean velocity of the stream, in feet per second; n is Kutter's coefficient of roughness; R, the hydraulic radius; and s, the slope or fall, in feet per foot. By transposition this becomes,

$$s = \frac{v^2 n^2}{(1.486)^2 R^{\frac{4}{3}}} \qquad (13)$$

The slope, s, is the height through which each unit of volume or mass of the flowing water must fall per foot of linear travel in order to provide the energy necessary to overcome friction. If it is assumed that the energy consumed in overcoming friction represents turbulent or eddy energy which is converted into heat, then it is evident that Equation (13) is a direct measure of the rate at which eddy energy is being transformed. On the other hand, if the slope in the reach under consideration is uniform, then the rate at which energy is being dissipated or transformed into heat is necessarily identical with that at which eddy energy is being produced in the channel. In other words, it is a measure of the rate at which energy is being impounded, so to speak, in eddy formation around the channel perimeter.

Since the velocity under the conditions assumed is known, with a known rate of inflow, and the hydraulic radius is also known, it follows that a function—such as Equation (13)—which apparently is directly proportional to the amount of energy of turbulence in the stream, can always be derived for any cross-section if the coefficient of roughness, n, is known. This coefficient can easily be determined, of course, from slope measurements in connection with the known rate of discharge.

This method of procedure should show a definite relation between the function indicated by Equation (13) and the error of mean velocity determined by any given type of meter at a given velocity in the stream. It is possible that the character of turbulence may differ in two different streams even when the eddy energy per unit volume is the same. Whether the effect of the resultant turbulence on the operation of current meters would or would not be the same in the two cases is a question which cannot be answered definitely except as the result of experiments.

Suggested Program of Experiments.—The possibility of developing an index of turbulence, the value of which could readily be determined in connection with current-meter measurements and applied to their correction, deserves most careful consideration. Such a correction, if established on a sound basis, is as necessary and as well justified as, for example, the correction of astronomical observations for refraction. In spite of the extensive,

although exceedingly unsatisfactory, literature regarding current-meter operation in turbulent water, the writer believes that much more could be accomplished along this line, by carrying out a campaign of experiments with the following technique and procedure.

With an experimental channel such as that used by the authors, for example, deliver an accurately known quantity of water into the channel in such a manner that it shall flow as smoothly and as nearly free from initial turbulence as possible. Next, induce artificial turbulence in the water by means of rotors similar, for example, to the propeller of an electric fan. These rotors could easily be arranged on the bottom and sides of the channel at points up stream from the measuring section so as to produce cross-currents simulating those produced naturally by eddies thrown off from the channel perimeter in the manner already described.

The rotors should be propelled by electric motors or otherwise in such a manner that their speed can be controlled and determined so that the amount of energy put into the water in producing eddies or turbulence can be measured accurately. Measurements of the velocity would then be made at a sufficiently large number of index points in the cross-section to give an accurate determination of the apparent velocity with each type of meter, with different known velocities in the channel, and with different known amounts of energy being expended in producing turbulence.

If it develops that the departure of the measured mean velocity from the true mean velocity by any given type of meter bears a definite functional relation to the two controlled variables—the mean velocity, and the amount of energy expended in producing turbulence—a start will have been made toward a quantitative solution of the problem.

In that event it would seem worth while to conduct experiments under field conditions in natural river channels. In the case of the natural river channel the procedure would necessarily involve measuring the flow at a cross-section down stream, for example, from a dam or reservoir, where the quantity of inflow to the reach containing the measuring section could be accurately controlled and determined, and where the coefficient of roughness, n, for the adjacent reach of the river could also be determined.

Turbulence in air is measured quantitatively and its characteristics recorded graphically by means of a Dines pressure tube anemometer. In experiments such as those suggested herein it would be a valuable adjunct to the results to have turbulence graphs taken by some type of Pitot-tube recorder. Possibly such an instrument might be attached directly to the current meter for experimental work. It is evident that either a means must be found for expressing turbulence quantitatively in terms of known variables and applying corrections to velocities measured by various types of current meters in turbulent water, or a considerable degree of uncertainty must be attached to measurements of stream velocity by current meters where the flow is markedly turbulent. The writer hopes that experiments along the lines indicated may be made forthwith.

Varying Velocity Effect.—The authors very properly subdivide the effect of turbulence on the operation of a current meter into two categories:

1.—The effect of varying velocities in the line of flow. This may briefly be called the "varying velocity effect". The expression, "line of flow", will here be used to indicate the direction of the current with no turbulence. It is also parallel with the direction of the axis of the meter as ordinarily used.

2.—The effect of currents approaching the meter at an angle to its axis, or at an angle to the normal line of flow, a condition which occurs in turbulent flow. This may be called broadly the "cross-current effect".

The second investigation by the authors consisted in determining the relation between the reading with the meter moved alternately backward and forward in the channel, and the reading of the same instrument held stationary. In these experiments the meter was suspended at the center of the cross-section, and velocities of 1, 2, 3, and 5 ft. per sec. were created in the channel by the operation of the gates. No baffles were used.

Again, referring to Fig. 12, it appears that each meter was swung back and forth parallel with the line of flow, through a range of 2 ft., by a wheel and crank, using a connecting rod 2 ft. in length. The abscissas are in terms of the ratio of the velocity of the crank-pin of the rotating wheel to the stream velocity.

The maximum rate at which the meter travels in swinging back and forth was a little greater than, but may be assumed for purposes of discussion to have been practically indentical with, the velocity of the crank-pin. The velocity of the meter relative to the water varied approximately from $v - c$ to $v + c$ and *vice versa*, in which, v is the stream velocity and c, the crank-pin velocity. The conditions, therefore, were closely similar to those which would exist in a stream containing eddies if the eddies passed the meter in succession in such a manner that the tangential component of the velocity of a filament of the eddy of a radius equal to the radius of the crank-pin was added to the stream velocity and if this was succeeded shortly by a second eddy, the opposite or retrograde-moving side of which passed through the center of the meter.

Following out the idea that the energy involved in turbulence is a controlling factor in the errors of registration of current meters in turbulent water, the following analysis has been made, assuming that the meter passes, successively, through the advancing and retrograding sides of eddies, v being the stream velocity in the line of flow, and c, the maximum tangential eddy velocity encountered by the meter.

Let it be assumed that the conditions are such that the actual velocity encountered by the meter while passing through a reach of flowing water of length, L, increases uniformly from $v - c$ to $v + c$, the velocity at any moment being,

$$v' = (v - c) + \frac{2\,c}{L}\,l \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

in which, l is the distance the meter has traversed from the initial point. The kinetic energy per unit volume of water encountered by the meter at any point is,

$$E' = \frac{w\,v'^2}{2\,g} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

The mean kinetic energy per unit volume of water encountered in traversing the length, L, is,

$$E_{\text{ave.}} = \frac{w}{2\,g\,L}\int_{v-c}^{v+c} v'^2\,d\,l \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

but $d\,l = \dfrac{L}{2\,c}\,d\,v'$. Substituting and integrating, Equation (16) becomes:

$$E_{\text{ave.}} = \frac{w}{4\,g\,c} \times \frac{v'^3}{3} = \frac{w}{2\,g} \times \frac{1}{6\,c}\left[(v+c)^3 - (v-c)^3\right] \ldots (17)$$

Expanding and simplifying,

$$E_{\text{ave.}} = \frac{w}{2\,g}\left[v^2 + \frac{c^2}{3}\right] \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Let $c = k\,v$; then,

$$E_{\text{ave.}} = \frac{w}{2\,g}\left(v^2 + \frac{k^2\,v^2}{3}\right) = \frac{w}{2\,g}\left(1 + \frac{k^2}{3}\right)v^2 \ldots\ldots (19)$$

The average kinetic energy per unit volume at the constant velocity, v, is,

$$E_v = \frac{w\,v^2}{2\,g} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

If, therefore, the velocity varies between the limits described in the distance, L, the meter will encounter excess kinetic energy in each cubic foot of water over that encountered with uniform velocity amounting to,

$$E_{\text{ave.}} - E_v = \frac{k^2}{3} \times \frac{w\,v^2}{2\,g} \ldots\ldots\ldots\ldots\ldots\ldots (21)$$

It appears, therefore, that with a velocity varying uniformly there is available an amount of kinetic energy to re-act on the meter greater than with equal mean but uniform velocity. The effect of the varying velocity, therefore, should be proportional to $\dfrac{k^2}{3}$, and should be equal to $\dfrac{k^2}{3}$ times a factor, f, which may or may not be a simple proportionality factor. Fig. 12 shows the percentage of over-registration for two types of small Price meter with varying velocities and a mean velocity of 2.0 ft. per sec. In the experiments the velocity did not vary uniformly from $v - c$ to $v + c$. This point will be considered later.

In Table 12, Column (1) shows the value of the crank-pin velocity ratio, $\dfrac{c}{v}$; Column (2), the kinetic energy excess over that for a uniform velocity, or the value of $\dfrac{k^2}{3}$, expressed as a percentage. Column (3) shows, for the small Price meter, the percentage excess of meter reading over that for uniform velocity; Column (4) shows the computed excess, using a value of the factor, $f = 0.60$; and Columns (5) and (6) give similar data for the U. S. Geological Survey improved Price meter. Observed excess meter readings in Columns (3) and (5) are taken directly from the Fig. 12. The values of f are derived by taking the average ratio of these quantities to the energy excess given in Column (2).

Fig. 35 offers a comparison with Fig. 12 for the two small Price meters with symbols showing the computed over-registration of these meters as given

in Table 12 for various values of *c*. The agreement of the computed with the observed values is practically perfect and is certainly within the limits of errors of observation except for the smaller values of *k*, for which, apparently, the friction or inertia of the meter rotor has a stabilizing effect and reduces the over-registration. Fig. 12 shows that, for the Mensing-Ott and the three-blade meters, the medium Ott meter, and the Hoff four-blade meter, the value of *f* is zero. For the small Ott and the Hoff three-blade meters, the value of *f* is about 0.033.

TABLE 12.—RELATION OF OBSERVED OVER-REGISTRATION OF PRICE METER IN A CURRENT OF VARYING VELOCITY, TO OVER-REGISTRATION COMPUTED FROM ENERGY EXCESS

$k = \frac{c}{v}$	Energy excess ratio, $\frac{E_{ave.} - E_v}{E_v} \times 100$	EXCESS METER READING, PERCENTAGES			
		Small Price		U. S. Geological Survey Improved Price	
		Observed	Computed, $f = 0.60$	Observed	Computed, $f = 0.54$
(1)	(2)	(3)	(4)	(5)	(6)
0.5	8.3	1.5	4.8	1.0	4.0
1.0	33.3	16.0	20.0	14.0	18.0
1.5	75.0	45.0	45.0	42.0	40.5
2.0	133.0	80.0	80.0	76.0	72.0
2.5	208.3	125.0	125.0	116.0	112.0
3.0	300.0	175.0	180.0	160.0	162.0

An interesting fact brought out by the preceding analysis is that the over-registration of a cup meter in varying velocity can be determined analytically and expressed rationally in terms of the excess kinetic energy of varying flow.

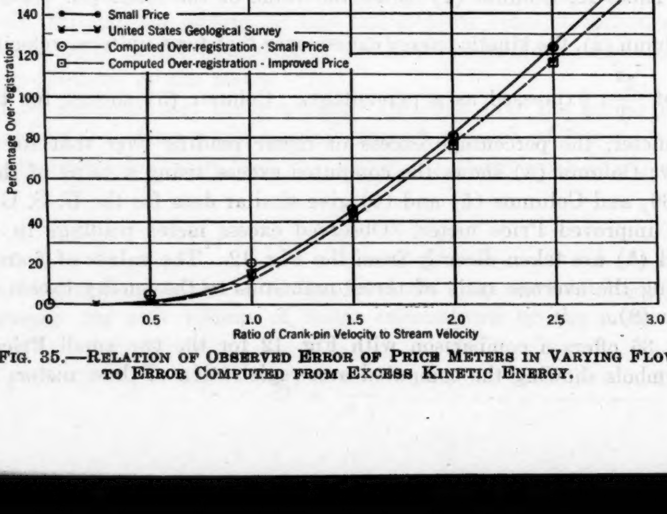

FIG. 35.—RELATION OF OBSERVED ERROR OF PRICE METERS IN VARYING FLOW TO ERROR COMPUTED FROM EXCESS KINETIC ENERGY.

The data indicate that f is a constant for a given meter and also that it is an important characteristic of the type of meter. It is apparent that the value of this constant can readily be determined for a given type of meter by the use of apparatus similar to that described by the authors. Some such standard apparatus should be developed and determinations of the factor, f, should be made in conjunction with the rating of meters which are to be used in turbulent water.

The authors' experiments were confined to a single current velocity, $v = 2$ ft. per sec., with various crank-pin velocities. It now appears that the results are applicable to any current velocity, providing the correction factor used is that applying to the proper value of $k = \frac{c}{v}$. In other words, the correction for over-registration of a Price meter, due to varying velocity, is the same for $v = 3$ and $c = 1.5$ as for $v = 2.0$ and $c = 1.0$, since $\frac{c}{v} = 0.50$ in both cases. Experimental verification of this point, however, is desirable. The preceding analysis is based on uniform velocity variation from $v - c$ to $v + c$.

Referring to Fig. 36, $a\,b$ is the velocity curve for uniformly varying velocity. It is evident that the velocity curve may follow an infinity of paths—one of which is shown by $m\,n$, Fig. 36—in changing from $v - c$ to $v + c$, for each of which the mean velocity will be v, the minimum, $v - c$, and the maximum, $v + c$.

If the velocity increases from $v - c$ to $v + c$ by n equal steps,

$$E_{\text{ave.}} = \frac{w}{2\,g\,n}\left[(v - c)^2 + \left(v - c + \frac{2\,c}{n-1}\right)^2 + \left(v - c + 2\,\frac{2\,c}{n-1}\right)^2 + \left(v - c + 3\,\frac{2\,c}{n-1}\right)^2 + \ldots \left(v - c + (n-1)\,\frac{2\,c}{n-1}\right)^2\right] \ldots\ldots (22)$$

or, since $c = k\,v$,

$$E_{\text{ave.}} = \frac{w\,v^2}{2\,g\,n}\left[(1 - k)^2 + \left(1 - k + \frac{2\,k}{n-1}\right)^2 + \left(1 - k + 2\,\frac{2\,k}{n-1}\right)^2 + \left(1 - k + 3\,\frac{2\,k}{n-1}\right)^2 + \ldots \left(1 - k + (n-1)\,\frac{2\,k}{n-1}\right)^2\right] \ldots . (23)$$

The term in brackets multiplied by $\frac{1}{n}$, is the ratio, $\frac{E_{\text{ave.}}}{E_v}$.

Let $a = 1 - k$, then when E_v is the kinetic energy corresponding to a uniform velocity, v, Equation (23) reduces to,

$$\frac{E_{\text{ave.}}}{E_v} = \frac{1}{n\,(n-1)^2}\left\{ a^2\,(n-1)^2 + [a\,(n-1) + 2\,k]^2 + [a\,(n-1) + 4\,k]^2 + [a\,(n-1) + 6\,k]^2 + \ldots [a\,(n-1) + (n-1)\,2\,k]^2 \right. \ldots\ldots\ldots (24)$$

Inspection of this equation shows that as n increases, the ratios of the first and last terms, to $(n - 1)^2$, remain constant, while the ratio of any intermediate term, to $(n - 1)^2$, decreases as n increases. There are n terms. Consequently, as n approaches infinity the average value of the terms will decrease. The minimum value will be that corresponding to a uniform change

of velocity, or the ratio of the excess kinetic energy to that for a constant velocity will approach asymptotically the value of this ratio for a uniformly changing velocity. The same result can be readily obtained in a particular case by numerical examples, using varying values of the number of steps, n, by which the velocity changes from $v - c$ to $v + c$. The general result may be stated in terms of the following theorem:

> In a mass of water having a mean velocity, v, and in which the velocities of different filaments vary between the values $v - c$ and $v + c$, the kinetic energy per unit volume is a minimum when the velocity changes uniformly from $v - c$ to $v + c$.

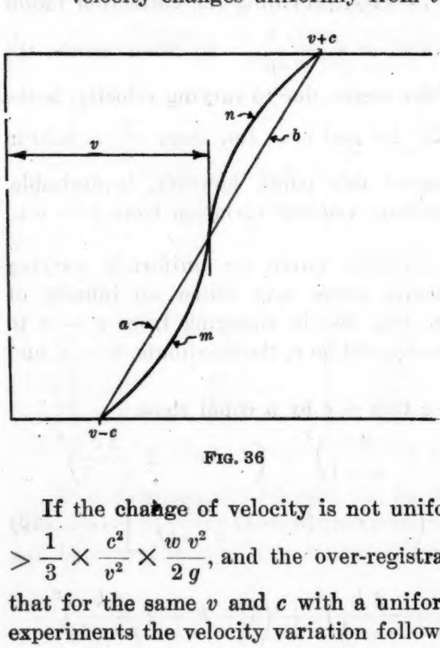

FIG. 36

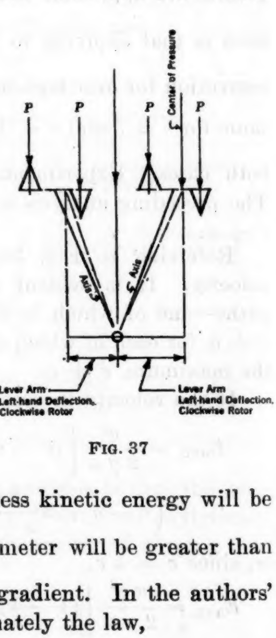

FIG. 37

If the change of velocity is not uniform, the excess kinetic energy will be $> \frac{1}{3} \times \frac{c^2}{v^2} \times \frac{w\,v^2}{2\,g}$, and the over-registration of the meter will be greater than that for the same v and c with a uniform velocity gradient. In the authors' experiments the velocity variation followed approximately the law,

$$v' = v - c \cos \frac{l}{L} \pi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

Hence, the excess energy was greater than $\frac{1}{3} \times \frac{c^2}{v^2} \times \frac{w\,v^2}{2\,g}$.

For the cosine velocity curve the total kinetic energy per unit volume is,

$$E = \frac{w}{2\,g} \int_0^L \left(v^2 - 2\,c\,v \cos \pi \frac{l}{L} + c^2 \cos^2 \pi \frac{l}{L} \right) d\,l \ldots\ldots (26)$$

Integrating and reducing,

$$E_{\text{ave.}} = \frac{w}{2\,g} \left[v^2 + \frac{c^2}{2} \right] = \frac{w}{2\,g} \left[1 + \frac{k^2}{2} \right] v^2 \ldots\ldots\ldots\ldots (27)$$

The excess kinetic energy in this case is $\frac{k^2}{2} \times \frac{w\,v^2}{2\,g}$, as compared with the value, $\frac{k^2}{3} \times \frac{w\,v^2}{2\,g}$, in case of uniform velocity variation.

Where the experiments are made with an apparatus that gives a cosine velocity variation as in the authors' experiments, the factor, f, should be computed on that basis. The values of f for two Price meters on the two different bases are as follows:

	Uniform change	Cosine law
Small Price.	0.60	0.40
U. S. Geological Survey Improved Price. . . .	0.54	0.36

It is perhaps immaterial what law of velocity change is used, provided a fixed standard is adopted for experimental purposes. Uniform change of velocity has a quality of simplicity and definiteness and gives the minimum excess energy per unit volume.

Referring to Fig. 12, the measurements taken by the Haskell meters are not comparable with those taken with the other meters for which results are shown on this diagram. The reader needs to be cautioned not to fall into the error of assuming that the Haskell meter, if used in the same manner as the other meters, would over-register—as the diagram indicates—to substantially the same extent as the Price meter. The contrary is the fact as shown at the end of the section entitled "Effect of Fluctuations in Speed of Flowing Water". Had the Haskell meter been operated to register every fifth or tenth revolution, as was the case with the other meters, it would have shown very little over-registration and would have given results agreeing with those for the Hoff and Ott meters.

One gains the impression from reading the text of the paper that the Price meter does not materially over-register due to varying velocities if the tangential component of eddy velocity (or the crank-pin velocity of the authors) is not greater than the mean stream velocity. This conclusion is not confirmed by Fig. 12, where it appears that with a crank-pin velocity equal to the stream velocity, the over-registration of the Price meter due to varying velocities alone equals 14 to 16 per cent. This is by no means negligible; in fact, it is sufficient to require most serious consideration of the validity of results obtained by the use of Price meters in highly turbulent water.

The writer believes that as a result of over-registration, measurements at higher stream stages by the U. S. Geological Survey with small Price meters are often materially in excess of the true discharges. Abundant proof of this can be adduced, but need not be presented here. To some extent this effect of over-registration is partly offset by the fact that most of the stream measurements at higher stages are made when the river stage is falling or after the crest of a flood has passed; the slope and, consequently, the velocity of the stream are, therefore, somewhat less than the normal slope and velocity for the same stream stage.

In view of the preceding, the writer cannot agree with the authors' conclusions that the meters tested will not be influenced by varying velocity of flow if up-stream currents are not encountered. Certainly, an error of 14 to 16%, as shown on Fig. 12 for the Price meter, with no up-stream velocity, does not confirm this conclusion.

Current-Angle Effect.—Meters of the cup type measure the effect of the current in the direction of its actual flow, whether or not they are held parallel to the line of flow. When operating in a cross-current such a meter with its axis parallel to the line of flow will measure approximately the actual velocity of the cross-current. If it measures correctly the velocity of the cross-current in the direction in which that current is flowing, then the error of the measurement (as distinguished from the error of the meter), will be the difference between the measured velocity and the product of the measured velocity by the cosine of the angle of the cross-current to the line of flow.

If, under these conditions, the meter does not correctly measure the actual velocity of the cross-current in its direction of flow, then the inherent error of the meter will be the difference between the actual velocity of the cross-current and the indicated velocity. If, as in the third series of experiments, the angle of the cross-current is known, its cosine value can be determined and the inherent error of the instrument can be found, as was done by the authors by comparing the velocity indicated by the meter with the true cosine value of the cross-current velocity. Meters of the cup type fail to some extent, and meters of the screw type to a greater extent, to measure the true velocity of the cross-current.

In practice, both varying velocity and cross-current effects are nearly always present together and the percentage error of the measured velocity is reflected by the product of the two percentage ratios of indicated to true velocities. If, due to varying velocity alone, the meter indicates a velocity of 115% of the true velocity and if, due to cross-current effect, the meter indicates a velocity of 95% of the cosine velocity, then for this particular angle of deflection the velocity as actually measured is 1.15×0.95, or 1.0925 times as great as the true velocity in the line of flow.

The authors found that the Price meter in particular showed a greater current-angle effect when the meter head was turned to the left than when turned to the right. A similar phenomenon was observed[9] by Charles P. Rumpf, M. Am. Soc. C. E., and also by E. H. Brown and Forrest Nagler,[10] Members, Am. Soc. C. E. These observations apparently establish the reality of the phenomenon. In the writer's experience, Price meters generally have clockwise rotation, but they are made for both clockwise and counter-clockwise rotation. The authors do not state in which direction the rotor revolved. Neither do they state the point at which, or the manner in which, the angle of deflection was measured. Presumably, it was measured by setting the meter at a certain angle before immersion.

Two possible causes of asymmetrical current-angle effects with Price meters are: (1) Deflection of the meter to a different angle than the measured angle, due to differential torsion of the suspension rod; and (2) a similar effect due to unbalanced dynamic pressure on the revolving rotor, something like that resulting from twirling a baseball in throwing a curve.

The first effect might result from the excess pressure on the cups over that on the cones or backs of the cups, as shown on Fig. 37, with the result that

[9] *Engineering News,* Vol. 17, 1914, pp. 1083–1084.

[10] *Proceedings,* Eng. Soc. of Western Pennsylvania, Vol. 30, 1914–15.

the lever arm of the torque on the suspension rod is greater when the meter is deflected in one direction than when deflected in the opposite direction, depending on whether the rotor has clockwise or counter-clockwise rotation. From the viewpoint of a scale model, the speed of a meter rotor and the velocity of a current in a fluid of the density of water, may be of the same order as the speed of rotation and velocity of a baseball in air.

Assuming that the meter is sufficiently rigid against torsion, the combined effect of these causes, if any, on the meter rotor, could readily be determined by experiments with a clockwise rotor with equal right and left-hand deflections of the meter, followed by identical experiments with a counter-clockwise rotor.

In this connection, it seems worth while to suggest that something might be added to the knowledge gained through experiments, such as those of the authors, on current-meter operation by studying the form and behavior of the water filaments passing the meter rotor.

This result could easily be accomplished with slight depths of submergence by suspending crystals of some soluble coloring matter, such as potassium permanganate, at intervals across the stream just up stream from the meter. This would provide colored streaks in the fluid acting on the meter without in any way interfering with the meter operation.

RICHARD VAN VLIET,[11] JUN. AM. SOC. C. E. (by letter).[11a]—During the thirty years, 1900 to 1930, numerous experiments have been performed by various parties in an endeavor to ascertain the errors resulting from the use of a current meter when subjected to various conditions of flow. A review of the results obtained indicates that the several parties practically agree on the errors caused by a meter when subjected to the various manipulated flows. The results obtained by experiments, such as those performed by Messrs. Yarnell and Nagler, are not very helpful to one who desires to correct actual field measurements made in perturbed waters.

A study of the results of these several experiments leads one to believe that measurements of perturbed water, made by a current meter, are subjected to considerable error. The writer believes that the error in measurement due to turbulent water has caused more consternation among the members of the Engineering Profession than is warranted. The best proof of this is the result obtained where simultaneous readings measured by a current meter are compared with those measured by a carefully calibrated weir, or sluice, or by volumetric means, etc. Volumetric measurements are extremely valuable and are far more applicable for correcting actual stream measurements of perturbed waters than experiments performed in laboratories with manipulated flows.

In contrast to the voluminous data obtained as the result of laboratory experiments with manipulated flows, there is a dearth of information available as to the relation that exists between discharges measured in the field by current meters and those measured by carefully calibrated weirs, sluices, etc. Probably the best series of experiments representing the comparison

[11] Asst. Engr., with Robert E. Horton, Cons. Hydr. Engr., Voorheesville, N. Y.

[11a] Received by the Secretary, March 12, 1931.

between the discharge measured by a current meter and that obtained from calibrated sluices, were those performed at the Assuan Dam on the Nile by Sir Murdoch MacDonald and H. E. Hurst.[12] The sluices at the Assuan Dam had previously been calibrated carefully by volumetric means so that the discharge through them could be computed with negligible error.

Forty experiments were performed, using either a small or a "medium"-sized Price meter. On inquiry, the type of instrument referred to as the "medium" sized meter was found to be that corresponding to the commonly known large sized Price meter with a 6-in. bucket wheel. Of these forty experiments, twenty-five were made with the small Price meter. Numerous velocity determinations were made for each vertical. As the sluices were carefully calibrated, a comparison between the discharges measured by the current meters and those given by the sluices will be a fairly good indication of the error encountered in measuring a stream by a Price meter. Table 13 is a summary of the results obtained by the small Price meter.

TABLE 13.—COMPARISON OF DISCHARGES AS MEASURED BY CURRENT METER AND BY SLUICE

Limit of discharge, in cubic meters per second	Number of tests	Average percentage of error*
Less than 700	2	+ 0.4
700 to 800	8	+ 3.5
800 to 1 000	4	+ 0.5
1 000 to 1 200	5	− 0.1
1 200 to 1 438	6	+ 1.7
Total	25	......

* Minus, when meter discharge is greater than sluice discharge.

The maximum under-registration for any individual test was 7.0%, with a discharge of 703 cu. m. per sec., whereas the maximum over-registration was 5.6%, with a discharge of 1 174 cu. m. per sec. The measurements made by current meter were performed in the natural stream channel in the vicinity immediately below the dam.

As in all work of this type some results are obtained, which deviate far from the average. It is only by performing a sufficient number of such experiments that an estimate of the average error can be reached. The question of correcting current-meter measurements of turbulent water does not permit the use of results obtained by small-scale models or manipulated canal currents. Data of the type similar to those obtained in connection with the experiments at the Assuan Dam are meager, and the writer feels that it is only by similar experiments that the mooted question of correcting current-meter measurements can be settled.

It is unfortunate that the authors have included only papers of investigations conducted in America in their bibliography (see Appendix). A paper by Mr. M. A. Hogan gives the results of numerous experiments performed in

[12] "Measurement of the Discharge of the Nile Through the Sluices of Assuan Dam," *Minutes of Proceedings*, Inst. C. E., Vol. CCXII, 1920–21, Part II, pp. 228–301.

Great Britain.[13] In general, the various results agree with those obtained by Messrs. Yarnell and Nagler. The general conclusions reached as a result of Mr. Hogan's studies and experiments were, as follows:

1.—The error in registration was dependent on the position of the meter with reference to the sides of the channel.

2.—The error in registration caused by the obliquity was greater in a small channel than in a large channel.

3.—The results showed a smaller error in registration for a shielded meter, such as the Stoppani, than for an Ott or an Amsler meter.

4.—Where considerable turbulence exists, a meter of the Price type will give results in excess of the true value. On the other hand, a meter of the screw type will give results which are deficient as compared with the true value.

The experiments relating to oblique flows were performed on Amsler, Ott, and Stoppani meters only, because the writer thought that the experiments performed by Brown and Nagler[14] on the Price meter were sufficient to establish the peculiarities relating to this type and did not warrant any duplication.

David L. Yarnell[15] and Floyd A. Nagler,[16] Members, Am. Soc. C. E. (by letter).[16a]—The many valuable suggestions for additional experimental work on current meters presented in the discussions reveal the unsettled status of the current meter, in the minds of many engineers, for water measurement under certain conditions. Although unable to proceed with this additional experimentation at this time, it is hoped that the work may be undertaken and reported upon in the near future. The specific suggestions which seem to warrant further investigation are:

(*a*) Comparative tests of flow measurements by the various meters with cable suspension in turbulent water.

(*b*) Further investigation of the cause for the difference in registration of the various meters in oblique flow approaching the meter from the right or left.

(*c*) Experiments on propeller shapes to develop a meter with the true cosine characteristic of rotation through a wide angle.

(*d*) Experiments with meters to determine the correction for turbulence along the line suggested by Mr. Leach.

(*e*) Experiments on meter registration with varying turbulence, following the suggestion of Mr. Horton concerning the rotating paddle; and,

(*f*) Comparative tests of the behavior of various meters near the water surface and sides and bottom of the channel.

It is with gratification that the writers have noted the comparisons by Messrs. Leach and Forrest Nagler, showing that their data checked exceptionally well with the results of similar tests performed by other experimenters.

[13] "River Gauging," by M. A. Hogan, Dept. of Scientific and Industrial Research, Lond., H. M. Stationery Office, 1925.

[14] *Proceedings*, Eng. Soc. of Western Pennsylvania, Vol. 30, 1914–15, pp. 280–323.

[15] Senior Drainage Engr., Bureau of Public Roads, U. S. Dept. of Agriculture, Iowa City, Iowa.

[16] Prof. of Hydr. Eng., Univ. of Iowa, Iowa City, Iowa.

[16a] Received by the Secretary, March 12, 1931.

The conditions of turbulence produced by means of paddles in the initial series of tests were probably more excessive than would ever be encountered in practice, as was surmised by Messrs. Nettleton, Leach, and Horton. Extreme conditions of turbulence were produced for the very purpose of exaggerating the discrepancies in registration of the various meters. It is a fact that the meters encountered sharp changes in speed and direction of flow. Eight points of measurement in a section under such violent disturbance were scarcely sufficient to establish a valid comparison between the average velocity recorded by the meters in comparison with the weir. However, in spite of this fact, it may be observed that with only one exception the meters listed in Table 2 arranged themselves in the same general order of accuracy in over-registration or under-registration as that indicated by the curves in Fig. 17, when the integration of the average deviation is computed in the manner suggested by Mr. Forrest Nagler.

In connection with these same tests, the impression that reliable rating curves were not available should be corrected. Recent curves were at hand for every meter tested, and values from these curves were used in the compilation of Table 2, which, therefore, should be quite accurate. The unusual turbulence is responsible for the variations in the listed values in Tables 1 and 2, since the ordinary period of observation (40 to 90 sec.) was insufficient to secure a true integration of the flow at each point (as suggested by Mr. Bennett and mentioned by the writers under "Constancy of Experimental Conditions"). Furthermore, the various meters had rotating elements of different diameters, and, in turbulent water, the large propeller might be actuated by a water filament of large diameter which had an average speed entirely different from the velocity of the smaller filament that strikes the meter with the small propeller, although the center of both propellers may have been at exactly the same point in the stream.

In performing their elementary experiments on meters with rod suspension, the writers purposely eliminated the possible variations that might have resulted when the meter was allowed the freedom of cable suspension. The performance of a meter with cable suspension would vary somewhat with different tail-pieces, and varying lengths of cable. With cable suspension, it is believed that the various meters would arrange themselves in practically the same order with regard to over- or under-registration, and as concluded by Mr. Leach:

> "The screw meter will not under-register as much with cable suspension as when rigidly suspended, owing to the tendency of all meters, including the screw types, to point into the current and so register the absolute velocity."

Some experimenters have found that the Price meter rotates only slightly slower with cable suspension than when rigidly held on a rod; hence, experiments with cable suspension would probably favor the screw meter more than the writers' experiments with rigid rod suspension have indicated.

Mr. Horton suggests some causes for the difference in registration of the Price meter when the current strikes at equal angles from the right or left, mentioning the fact that the experiments of C. P. Rumpf, M. Am. Soc. C. E.,

TABLE 14.—PHYSICAL CHARACTERISTICS OF CURRENT METERS USED IN TESTS

Name of meter	Meter number	Fig.	Type of Propeller: Screw or cup	Blades: Number	Blades: Kind	Number of revolutions per contact	Linear* distance for one revolution of propeller, in feet	Outside diameter of propeller, in inches	Distance, center of support to center of wheel, in inches	Direction of rotation of wheel ‡
Hoff	9	3a	Screw	3	Rubber	10	0.94	....	......	Clockwise
Hoff	9	3b	Screw	4	Rubber	10	0.94	4.00	......	Clockwise
Ott-Small	2914	4a	Screw	3	Fan	50	0.41	2.30	4.59	Counter-clockwise
Ott-Medium	2908	4b	Screw	2	Conical	25	0.85	4.79	7.43†	Counter-clockwise
Ott-Mensing	3246	5a	Screw	2	Conical	25	1.71	6.80	12.10†	Counter-clockwise
Ott-Mensing	3246	5b	Screw	3	Cylindrical	25	1.72	7.04	11.75	Counter-clockwise
Haskell-Type B	154	6	Screw	4	High pitch	1	3.02	7.77	13.15†	Clockwise
Haskell-Type B	154	7	Screw	4	Low pitch	1	1.23	7.67	12.98†	Clockwise
Price-Small	211880	2	Cup	6	Cones	5	2.23	5.00	3.90	Counter-clockwise
Price-Acoustic	19	1	Cup	6	Cones	10	2.24	5.00	0	Counter-clockwise
Price-Small improved	26295	8	Cup	6	Cones	5	2.24	5.00	......	Counter-clockwise

* At stream velocities of 2 ft. per sec.

† Distance measured to up-stream end of wheel

‡ Looking downward or down stream at meter

and of Mr. Forrest Nagler, as well as those of the writers, establish the reliability of these phenomena. The curves for the Pegram screw type of meter described by Major Somervell also showed this same characteristic, which was likewise observed by the writers on other meters with propellers of the screw type. The phenomenon seems to result from some unsymmetrical interference of the current leaving the rotating propeller with the frame of the meter. For some meters the magnitude of the deviation was different at different velocities. Local disturbance, as suggested by Mr. Leach, may possibly have been responsible for some of the deviation, although tests performed with current flowing in the main canal (which was 10 ft. wide) also showed similar differences between the registration of currents from the right and left direction. However, many tests in which screw meters have been towed in still water, have shown no great deviation.

In response to requests for additional information with regard to the meters used by the writers, Table 14 is presented giving physical characteristics of the various meters.

Numerous tests in which measurements by current meters have been checked by flow determinations by other methods have been made during the ten years, 1920 to 1930, in the hydraulic canal at the Laboratory of the State University of Iowa. When the conditions have been favorable and care has been taken to traverse the measuring section adequately, the measurement of flow with current meters has always proved exceedingly reliable. In connection with experiments on the flow through box culverts in 1922 to 1924, one hundred and ten discharge measurements were made by Mr. Yarnell of flows between 5 and 165 cu. ft. per sec., by means of a Pitot tube, the small Ott current meter, and a sharp-crested rectangular weir.

More than one-half the comparisons showed less than 2% variation between the measurements by the weir and those by the current meter, and most of the measurements that showed a greater departure were performed under conditions not entirely ideal for the best current-meter measurement. However, in presenting the paper, the writers decided that it was advisable to compare the various meters purely on the basis of their registration of current velocity, rather than to compare the measured quantities of flow. In the latter case the location and number of metering points, the method of computing the mean velocity, and the measurement and method of computation of area of cross-section, often introduce other possible errors of no small magnitude.

The writers were greatly interested in Mr. Groat's discussion of rules for obtaining the average velocity in a vertical plane from velocity readings at definite points. The analysis and results are similar to those suggested by Patnutij Tchebycheff. The work of Tchebycheff was brought to the writers' attention in 1925 by A. Streiff, M. Am. Soc. C. E., in an unpublished paper entitled "The Use of Tchebycheff's Formula in Current Meter Measurements."[17] The formula has been derived by the Naval Architect, J. H. DeHeer. A comparison of the ordinates derived by Mr. Groat (see Table 9) with those obtained by Tchebycheff is given in Table 15. The results for two.

[17] *de Ingenieur*, November 14, 1908; see, also, *Hütte*.

three, and five ordinates are identical. The writers have tested the Tchebycheff rule on a number of mathematical curves and generally have found sufficiently close agreement to the true average for accurate practical work.

TABLE 15.—POINTS FOR OBTAINING THE MEAN OF A VERTICAL VELOCITY CURVE

Number of points in vertical	LOCATION OF POINTS IN THE VERTICAL, IN FRACTION OF TOTAL DEPTH	
	P. Tchebycheff	B. F. Groat
2	0.211825	0.211325
	0.788675	0.788675
3	0.146446	0.146446
	0.500000	0.500000
	0.853554	0.853554
4	0.102673	0.1127017
	0.406204	0.3709006
	0.593796	0.6290994
	0.897327	0.8872983
5	0.083751	0.0837513
	0.312730	0.3127293
	0.500000	0.5000000
	0.687270	0.6872707
	0.916249	0.9162487
6	0.066876	0.07920706
	0.288740	0.2475242
	0.366682	0.4158414
	0.633318	0.5841586
	0.711260	0.7524758
	0.933124	0.9207929
7	0.058069	....
	0.235672	....
	0.338044	....
	0.500000	....
	0.661956	....
	0.764328	....
	0.941931	....
9	0.044206	....
	0.199490	....
	0.235619	....
	0.416047	....
	0.500000	....
	0.583953	....
	0.764381	....
	0.800510	....
	0.955794	....

To average the measurements made by two different types of meters (the Ott and the Price, for example) appears now (1931) to be the "only rational method" of obtaining a reliable measurement of the discharge of turbulent water. This suggestion made by Mr. Leach follows the proposal[18] made by Mr. Groat, and has been specified in the A. S. M. E. Power Test Code, for measuring water by current meters in the testing of hydro-electric plants. The curves presented by the writers are useful in the determination of the comparative over- or under-registration of the meters being used. In extremely turbulent water, the use of a single meter does not appear feasible until some method is devised for determining an index of the relative degree of turbulence.

[18] *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1231.

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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

DISCUSSIONS

ANALYSIS OF CONTINUOUS FRAMES BY DISTRIBUTING FIXED-END MOMENTS

Discussion

BY MESSRS. T. F. HICKERSON, F. H. CONSTANT, W. N. DOWNEY, AND E. C. HARTMANN

T. F. HICKERSON,[11] M. AM. SOC. C. E. (by letter).[11a]—The author has presented an ingenious method for determining bending moments in continuous frames, which, simple as it may seem, is in effect an application of the slope deflection method through the medium of successive arithmetical approximations instead of the formal solution of algebraic equations. Although no mathematics beyond arithmetic is used, yet there are many dangerous pitfalls incident to an attempted application of this method by any one who is unversed in the basic principles of continuity of elastic structures.

The method is of most value in analyzing unsymmetrical frameworks (sidesway neglected), such as that given by the author in his Fig. 1, but for less complicated cases—the kind that ordinarily occurs in practice—it appears to be too much of a juggling process for general acceptance in a design office.

Believing that the use of coefficients offers the simplest and quickest way of obtaining moments and shears at various sections in continuous beams and frames, the writer has prepared a comprehensive set of tables for this purpose. A simple outline, with a sample table of coefficients, was presented in connection with another paper.[12]

Four cases will now be cited illustrative of typical frameworks and their analyses by two methods: (1) The method of "Tabular Coefficients", proposed by the writer; and (2), the method of "Carry-Over Moments", proposed by the author.

NOTE.—The paper by Hardy Cross, M. Am. Soc. C. E., was published in May, 1930, *Proceedings*. Discussion of the paper has appeared in *Proceedings*, as follows: September, 1930, by Messrs. C. P. Vetter, L. E. Grinter, S. S. Gorman, A. A. Eremin and E. F. Bruhn; October, 1930, by Messrs. A. H. Finlay, R. F. Lyman, Jr., R. A. Caughey, Orrin H. Pilkey, and I. Oesterblom; November, 1930, by Messrs. Edward J. Bednarski, S. N. Mitra, Robert A. Black, and H. E. Wessman; January, 1931, by Messrs. Jens Egede Nielsen, F. E. Richart, and William A. Oliver; February, 1931, by Messrs. R. R. Martel and Clyde T. Morris; and March, 1931, by Francis P. Witmer, M. Am. Soc. C. E.

[11] Prof. of Civ. Eng., Univ. of North Carolina, Chapel Hill, N. C.

[11a] Received by the Secretary, January 23, 1931.

[12] "Continuous Beams Over Three Spans," by I. Oesterblom, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 93 (1929), p. 1405.

Case 1.—Symmetrical, Two-Legged Bents.—The following notation[13] applies to this discussion:

l = length of beam, $A B$.
I_1 = moment of inertia of beam, $A B$.

S_1 = stiffness of beam, AB, $= \dfrac{I_1}{l}$.

h = length of column, $A C$ (or $B D$).
S = stiffness of column, $A C$ (or $B D$).

r = relative column stiffness $= \dfrac{S}{S_1} = \dfrac{I}{I_1} \dfrac{l}{h}$.

$M_A = M_{AB}$ = resisting moment at End A of Member AB (or AC).

The moment, C_{AB}, equals that part of M_{AB} which depends only on the loads; if the ends, A and B, are fixed on the same levels, $M_{AB} = - C_{AB}$.

Referring to the frame shown in Fig. 36, where the columns are fixed at the bases,

$$M_A = -\frac{1}{2}\left[(C_{AB} + C_{BA})\left(\frac{2\,r}{1 + 2\,r}\right) + (C_{AB} - C_{BA})\left(\frac{r}{6 + r}\right)\right] \text{.(42)}$$

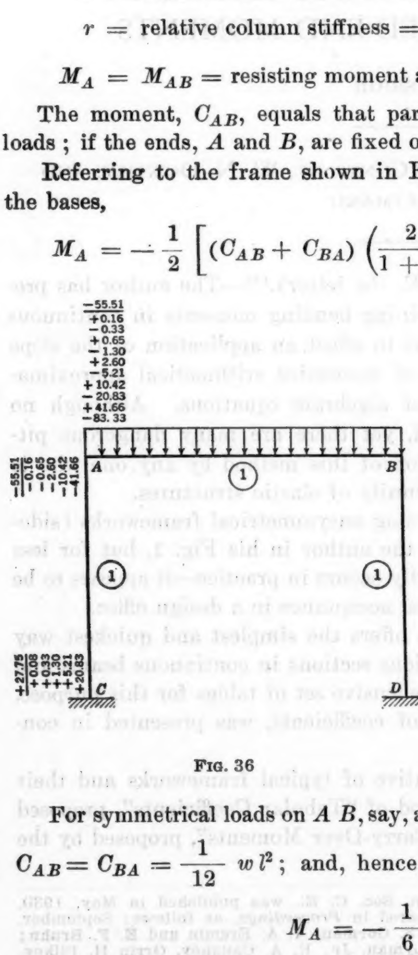

FIG. 36

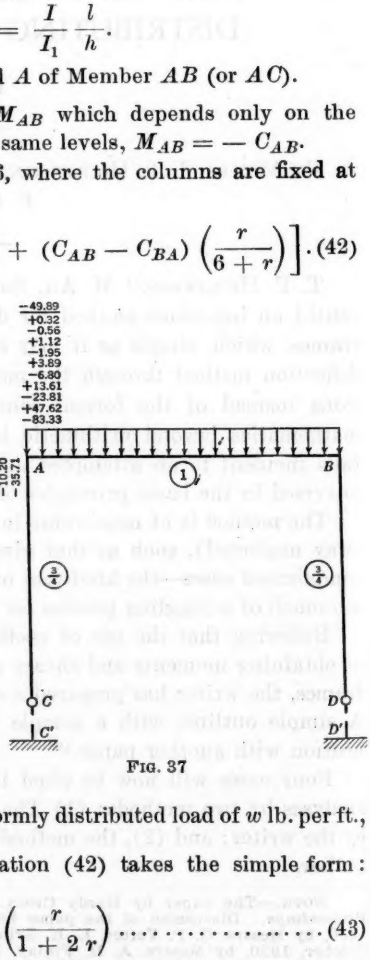

FIG. 37

For symmetrical loads on $A B$, say, a uniformly distributed load of w lb. per ft., $C_{AB} = C_{BA} = \dfrac{1}{12}\, w\, l^2$; and, hence, Equation (42) takes the simple form:

$$M_A = -\frac{1}{6}\, w\, l^2 \left(\frac{r}{1 + 2\,r}\right) \ldots\ldots\ldots\ldots\ldots (43)$$

Referring to Fig. 37 where the columns are hinged at the bases,

$$M_A = -\frac{3}{2}\,(C_{AB} + C_{BA})\left(\frac{r}{2 + 3\,r}\right) \ldots\ldots\ldots\ldots (44)$$

---

[13] This notation is the same as that used in *Bulletin 108*, Univ. of Illinois Eng. Experiment Station.

For a symmetrical loading on $A\,B$, say, a uniform load of w lb. per ft.,

$$M_A = -\frac{1}{4}\,w\,l^2\left(\frac{r}{2+3\,r}\right)\dots\dots\dots\dots\dots\dots(45)$$

Tables may be constructed based on Equations (42) to (44), inclusive, such that the corner moments for any vertical loading and for any condition of relative stiffness may readily be determined. Furthermore, such tables enable one to compare by inspection the difference between the effects of fixed and hinged terminals.

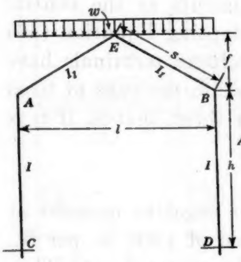

FIG. 38

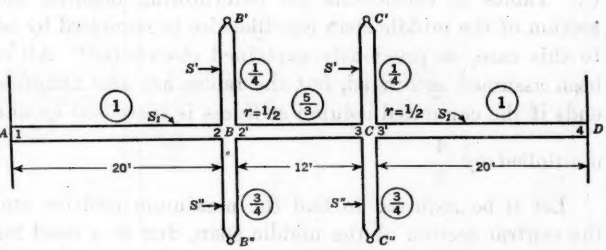

FIG. 39

The author's method as applied to the fixed frame is indicated in Fig. 36. Although the assumed conditions are about the simplest possible ($r = 1$ and a symmetrical load of 1 000 lb. per ft.), the same results are obtained more easily by means of Equation (43), or by means of tabular coefficients prepared from it.

In applying the author's method to the hinged frame of Fig. 37, the writer found it quite tedious to carry moments out to the hinge at Point C and then balance the moment and carry it back again. This procedure was abandoned, and computations were made on the basis of columns three-fourths as stiff, just as if fixed terminals existed at Points C' and D'. This possibility was pointed out by Professor Cross under the caption "Variations of the Method". Again, it may be said that the desired results could be obtained much more readily by applying Equation (45), or from tabular coefficients determined from it.

Case 2.—On Arch Frames.—The frame shown in Fig. 38 is subjected to a uniformly distributed load of w lb. per ft. A table may be used which is based on the formulas derived from the least work theorem; it would give coefficients for determining the corner moments,

$$M_A = \phi\,w\,l^2\dots\dots\dots\dots\dots\dots(46)$$

Thus, if $l = 30$ ft., $h = 20$ ft., $f = 6$ ft., $r = \frac{1}{2}$, $\frac{f}{h} = 0.3$, and $w = 1\,000$ lb. per ft., the values of ϕ given by the table (not given herewith) are -0.050 and -0.052 for the hinged and fixed frames, respectively. For the hinged frame, $M_A = -0.050 \times 1\,000 \times (30)^2 = -45\,000$ ft-lb.; and for the fixed frame, $M_A = -0.052 \times 1\,000 \times (30)^2 = -46\,800$ ft-lb. In Fig. 38, $S = \frac{I}{h}$; $S_1 = \frac{I_1}{S}$; and $r = \frac{S}{S_1} = \frac{I}{I_1}\,\frac{s}{h}$.

If the entire load is concentrated at the central ridge (at Point *E*, Fig. 38), the foregoing values would be multiplied by $\frac{3}{2}$, or increased by 50% ; and for other symmetrical loadings, the corresponding moments are obtained by merely multiplying the coefficients in the prepared table for this case, by known ratios.

The author's method seems inapplicable to the arched frame; in fact, the slope reflection method itself is hardly adaptable to this case.

Case 3.—Three Continuous Beams.—Fig. 39 shows a symmetrical arrangement of three continuous beams rigidly connected to columns at *B* and *C*. Tables of coefficients for determining bending moments at the central section of the middle span can likewise be compiled by adapting Equation (46) to this case, as previously explained elsewhere.[14] All column terminals have been assumed as hinged, but the tables are also adaptable to the case of fixed ends if the computed column stiffness is increased by one-third, that is, if it is multiplied by $\frac{4}{3}$.

Let it be required to find the maximum positive and negative moment at the central section of the middle span, due to a dead load of 1 200 lb. per ft., and a live load of 1 000 lb. per ft. Furthermore, let it be assumed: (1) That the two end spans are each 20 ft., and the middle span is 12 ft.; (2) that the beam terminals are one-half fixed (that is, the terminal moments are one-half the values for absolutely fixed conditions); and (3) that the ratio of average column stiffness to that of the end beam is $\frac{1}{2}$.

Referring to the proper table of coefficients (not given herewith), the following values of ϕ are obtained; -0.017 for first span loaded; $+0.026$ for second span loaded; and -0.017 for third span loaded; from which the following moments are easily computed;

Total maximum negative moment = − 17 440 ft-lb.
Total maximum positive moment = + 6 560 ft-lb.

To solve this problem by means of the author's method is a longer and more tedious process than that of tabular coefficients; so much so, that the writer got no farther than to find the moments (at the supports) due to the live load of 1 000 lb. per ft. on the middle span when the beam terminals are fixed and when they are hinged. The problem is then only about one-third solved, since the moment at the middle of the span must be determined from the equations of statics after the moments at the supports and the reactions are known.

Case 4.—Multiple Spans.—Consider a multiple-span arrangement, as, for example, the five continuous spans shown in Fig. 40(*a*). The writer has computed shear and moment coefficients upon the following bases: (1) The moment at any interior support and the shear at either side of it are greatest when the two spans adjacent to that support are covered with live and dead load, while the next span to the right and to the left is covered only with dead load; (2) the effect of loads on remote spans; that is, beyond the two spans from the support, in either direction, is neglected; and (3) the span arrangement beyond any support may be removed if a certain fixation factor is substituted therefor.

[14] *Transactions*, Am. Soc. C. E., Vol. 93 (1929), Table 12, p. 1407.

In order to find the maximum value of the bending moment, M_2, at the right end of the member, $B\,C$ (Fig. 40(*a*)), two sets of loads will be dealt with: (1) Live and dead load on the two spans, $C\,B$ and $C\,D$, adjacent to the support, C (Fig. 40(*b*)); and (2), dead load on the spans, $B\,A$ and $D\,E$ (Fig. 40(*c*)).

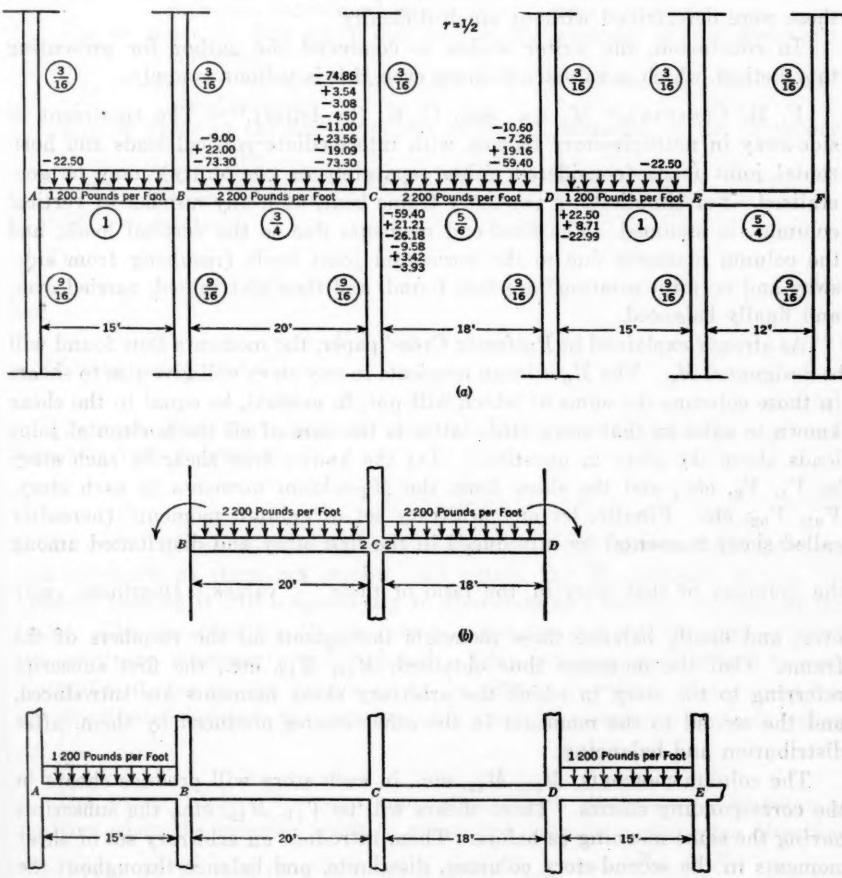

FIG. 40

Referring to a table (not given herewith), with $x = \dfrac{18}{20} = 0.9$:

$$M_2 = [-0.117 + 0.29\,v + 0.010\,t]\,w\,l^2 \ldots\ldots\ldots\ldots (47)$$

in which, v and t are the fixation factors at the ends, B and D, in Fig. 40(*b*); that is, $v = 1$ and $t = 1$, if both ends are fixed. For this particular span arrangement, $v = 0.65$ and $t = 0.70$, hence, $M_2 = -0.091 \times 2\,200 \times (20)^2 = -80\,080$ ft-lb.

Referring to a table (not given herewith) for the proper coefficient to use in Fig. 40(*c*), the value of 0.019 is obtained; hence, $M_2 = + 0.019 \times 1\,200 \times (15)^2 = + 5\,130$ ft-lb. Combining the two values of M_2 thus obtained, and the total maximum moment equals $M_2 = - 80\,080 + 5\,130 = - 74\,950$ ft-lb.

In applying the author's method to this problem, only those carry-over moments actually needed in evaluating M_2 are indicated in Fig. 40(*a*); and these were determined without much difficulty.

In conclusion, the writer wishes to commend the author for presenting this method, which is valuable to know even if it is tedious to apply.

F. H. CONSTANT,[15] M. AM. SOC. C. E. (by letter).[15a]—The treatment of side-sway in multiple-story frames, with intermediate vertical loads and horizontal joint loads (considered either separately or conjointly), may be generalized. For simplicity, a two-story frame bent, with any number of vertical columns, is assumed. The fixed-end moments due to the vertical loads, and the column moments due to the horizontal joint loads (resulting from side-sway and no joint rotation) are first found, and then distributed, carried over, and finally balanced.

As already explained in Professor Cross' paper, the moments thus found will be designated M_0. The M_0-column moments in any story will give rise to shears in those columns the sums of which will not, in general, be equal to the shear known to exist in that story (this latter is the sum of all the horizontal joint loads above the story in question). Let the known true shear in each story be V_1, V_2, etc., and the shear from the M_0-column moments in each story, V_{01}, V_{02}, etc. Finally, let any arbitrary set of column moments (hereafter called shear moments) be introduced in the first story and distributed among the columns of that story in the ratio of their $\frac{I}{h^3}$ values. Distribute, carry over, and finally balance these moments throughout all the members of the frame. Call the moments thus obtained, M_{11}, M_{12}, etc., the first subscript referring to the story in which the arbitrary shear moments are introduced, and the second to the moments in the other stories produced by them, after distribution and balancing.

The column moments, M_{11}, M_{12}, etc., in each story will produce shears in the corresponding stories. These shears will be V_{11}, M_{12}, etc., the subscripts having the same meaning as before. Then, introduce an arbitrary set of shear moments in the second-story columns, distribute, and balance throughout the frame as before. Call these moments M_{21}, M_{22}, etc., and the corresponding shears V_{21}, V_{22}, etc.

Stating this nomenclature more concisely:

M = required correct moment in any member.
V_1 = shear known to exist in the first-story columns.
V_2 = shear known to exist in the second-story columns.
M_0 = distributed and balanced moments, due to the fixed-end moments in the beams, and the shear moments in the columns, produced by the external loading.

[15] Prof. of Civ. Eng., Princeton Univ., Princeton, N. J.

[15a] Received by the Secretary, February 16, 1931.

V_{01} = shear in the first story, due to the M_0-moments in the first-story columns.

V_{02} = shear in the second story, due to the M_0-moments in the second-story columns.

M_1 = distributed and balanced moments due to an arbitary set of shear moments introduced in the first-story columns.

M_2 = distributed and balanced moments due to an arbitrary set of shear moments introduced in the second-story columns.

V_{11} = shear in first story due to the M_1-moments in the first-story columns.

V_{12} = shear in the second story due to the M_1-moments in the second-story columns.

V_{21} = shear in the first story due to the M_2-moments in the first-story columns.

V_{22} = shear in the second story due to the M_2-moments in the second-story columns.

C_1 and C_2 = constants.

Then,

$$V_1 = V_{01} + C_1\, V_{11} + C_2\, V_{21} \qquad (48)$$

and,

$$V_2 = V_{02} + C_1\, V_{12} + C_2\, V_{22} \qquad (49)$$

Solving for C_1 and C_2,

$$M = M_0 + C_1\, M_1 + C_2\, M_2 \qquad (50)$$

For a one-story bent subjected only to a horizontal force at the top joint, Equation (50) reduces to,

$$M = M_0\, \frac{V_1}{V_0} \qquad (51)$$

In a multiple-story building there will be as many independent equations and constants as there are stories. To obtain the M_0-moments due to the external loading it will be necessary to distribute and balance throughout the frame. The moments, M_1, M_2, etc., are only great in and near the story in which the arbitrary moments are introduced. They decrease rapidly and become insignificant one or two stories away, in either direction. Shear moments are distributed between the columns of a story in proportion to their values of $\frac{I}{h^3}$.

The writer prefers the sign convention which designates a moment as positive when the member tends to rotate the joint in a clockwise direction. The shear at the foot of a column, $A\ B$, is, then, $V = \frac{M_A + M_B}{h}$, the sign and direction of V being automatically controlled by the signs of the numerical values of M_A and M_B. The work is also greatly expedited by balancing first the joints having the largest unbalanced moments, and carrying over the distributed moments at once, before proceeding to the next joint.

W. N. Downey,[16] Assoc. M. Am. Soc. C. E. (by letter).[16a]—The design of a rigid frame by the conventional methods generally in use is unquestionably a tedious and laborious process, because a clear conception of the frame action

[16] Care, Cincinnati Union Terminal Co., Cincinnati, Ohio.

[16a] Received by the Secretary, March 7, 1931.

is obscured by a maze of algebraic manipulations. A method by which the designer can visualize step by step exactly what is happening is of great value. For this reason Professor Cross has made a valuable contribution to the literature on the subject of the statically indeterminate frame.

The problems introduced in the paper may also be analyzed by successive corrections to the angular turns at the ends of the members. This method, as developed by the writer, has been found to be very rapid. The method of procedure may be stated briefly. All the angular turns, due to the load, are calculated approximately, then corrections are made to these approximate values to obtain more nearly the correct angles. As many corrections may be calculated as is considered necessary for the problem under consideration. From the magnitude of the corrections it can be judged when the computations have been carried far enough. The exactness of the final angles depends only on how many corrections the designer cares to make. The second correction generally gives results exact enough for ordinary design purposes.

Besides being rapid the method of successive corrections has other distinct advantages. It is particularly well suited to the calculation of influence lines for moving loads, which are almost indispensable in bridge design; the effect of horizontal deflection (and, consequently, temperature changes) and of horizontal loads at the joints is taken into consideration in a simple and direct manner. In addition, the method lends itself well to tabular arrangement of the computations, thus minimizing the chance of error in the numerical work. It applies equally as well to beams of variable moments of inertia as to beams of constant section.

Calculations Neglecting the Effect of Horizontal Deflection.—The slope-deflection equations[17] give a convenient way of visualizing the constants required in the approximations for the angles. These equations may be written:

For a beam restrained at both ends:

$$M_{AB} = E\,K_{AB}\left[\beta_{AB}\,\theta_A + \beta'_{AB}\,\theta_B - \beta''_{AB}\,\frac{d}{h_{AB}}\right] \mp C_{AB} \ldots\ldots (52)$$

$$M_{BA} = E\,K_{AB}\left[\beta_{BA}\,\theta_B + \beta'_{BA}\,\theta_A - \beta''_{BA}\,\frac{d}{h_{AB}}\right] \pm C_{BA} \ldots\ldots (53$$

For a beam restrained at A and fixed at C:

$$M_{AC} = E\,K_{AC}\left[\beta_{AC}\,\theta_A - \beta''_{AC}\,\frac{d}{h_{AC}}\right] \mp C_{AC} \ldots\ldots\ldots\ldots (54)$$

$$M_{CA} = E\,K_{AC}\left[\beta'_{CA}\,\theta_A - \beta''_{CA}\,\frac{d}{h_{AC}}\right] \pm C_{CA} \ldots\ldots\ldots\ldots (55)$$

For a beam restrained at A and hinged at D:

$$M_{AD} = E\,K_{AD}\,\beta_{AD}\left[\theta_A - \frac{d}{h_{AD}}\right] \mp H_{AD} \ldots\ldots\ldots\ldots\ldots (56)$$

In Equations (52) to (56), $\beta + \beta' = \beta''$; $\beta'_{AB} = \beta'_{BA}$; and $K = \frac{I}{h}$.

For the case of a beam of constant section: $\beta_{AB} = \beta_{BA} = \beta_{AC} = 4$; $\beta'_{AB} = \beta'_{BA} = \beta'_{CA} = 2$; $\beta''_{AB} = \beta''_{BA} = \beta_{AC} = \beta''_{CA} = 6$; and, $\beta_{AD} = 3$.

[17] *Bulletin 108*, Univ. of Illinois, Eng. Experiment Station.

The values of β and the fixed end moments for beams of varying moment of inertia may be found by any of the standard methods of calculation, or may be readily calculated from the tables[18] given by Strassner, or from the tables[19] given by Walter Ruppel, Assoc. M. Am. Soc. C. E. Both writers cover a wide range of conditions.

Sign Conventions and Notation.—A positive angular change is measured from the original position of the axis in a clockwise direction about the end of the member considered. A positive deflection is measured in the same direction as a positive angular change. An external moment acting in a clockwise direction about the end of a member is positive. An external load acting in a clockwise direction about a joint is negative. The following notation is used:

θ = change in slope of the tangent to the elastic curve at the end of a member, the subscript denotes the joint; θ_A = the change in slope at A.

d = relative displacement of the ends of a member with respect to the original direction of the member.

K = moment of inertia of the member divided by its length.

θ' = change in slope of the tangent to the elastic curve at the end of a member when no horizontal movement is considered, the subscript denotes the joint. θ'_A = the change in slope at A.

β, β', and β'' = constants determined by the elastic properties of the member.

α = angular turn at the joints produced by the deflection, $d = 1$. The subscript denotes the joint. α_A = the rotation at A.

H = horizontal reaction on the frame produced by the deflection, $d = 1$.

ρ = angular turn at the joints produced by $H = -1$. The subscript denotes the joint. ρ_A = the rotation at A.

d_A = deflection of the top of the columns due to a moment, $- X_a$. (The numerical value of $d_A = \rho_A$). Likewise, for d_B, d_C, etc.

X_a = external moment applied at the joint, A. Similarly, for X_b, X_c, etc.

Z_a = angular turn at A due to $X_a = -1$. All other joints held fixed. Similarly, for Z_b Z_c, etc.

X_{ba} = external moment at B, due to a unit angular turn at A, when Joint B is fixed against rotation. Similarly, for X_{ab}, X_{cb}, etc.

$X_{a(d)}$ = external moment about the end, A, of a member, due to the deflection, $d = 1$, when the end, A, is fixed against rotation, and the other end is either fixed or hinged.

Procedure.—The fundamental conception of calculating the angular turns by successive corrections is similar to that of distributing end moments. Consider one joint at a time as elastic and calculate the angular change at this elastic joint, due to the applied external load. Then proceed to the next joint, which is now considered as the only elastic joint, and calculate the angular turn at this joint, due to the angular change at the joint just calculated and the applied external load. In this manner, calculate angular turns at each joint in the structure and obtain the first approximation, $\Delta\,\theta'$.

18 "Neuere Methoden," Vol. I.

19 *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 152.

For instance, consider the joints, A, B, C, and D, in sequence and calculate the first approximations, $\Delta\,\theta'_A$, $\Delta\,\theta'_B$, $\Delta\,\theta'_C$, and $\Delta\,\theta'_D$. To avoid confusion as to whether all effects have been considered, begin at the left joint and calculate to the right, or *vice versa*.

To obtain the corrections, θ'_1, begin with the last calculated $\Delta\,\theta'$ (for instance, $\Delta\,\theta'_D$), and compute the angular turn at C (θ'_{1C}), due to $\Delta\,\theta'_D$. The sum, $(\Delta\,\theta'_C + \theta'_{1C})$, would be a closer approach to the true value of θ'_C. Proceed to Joint B and calculate the angular turn, θ'_{1B}, due to angular change, $(\Delta\,\theta'_C + \theta'_{1C})$. Continue in this manner through the frame. These values constitute the first corrections, θ'_1.

Determine θ'_2 in the same manner as θ'_1, using only those increments of θ' which have not been previously used in going through the frame in this direction. That is, θ'_{2B} is obtained by calculating the angular turn at B due to θ'_{1A}; θ'_{2C} is obtained by computing the angular turn at C due to $(\theta'_{1B} + \theta'_{2B})$, etc.

All other corrections are made in the same way as θ'_2. When, from the magnitude of the corrections, it is seen that the angles are exact enough for the problem considered, sum up the first approximation and the corrections for each angle; that is:

$$\theta' = \Delta\,\theta' + \theta'_1 + \theta'_2 + \ldots \qquad (57)$$

Angle θ' is the rotation of the joint due to the effect of the external loads, the effect of horizontal deflection being neglected.

Computations of Constants for Determining Angular Changes.—Referring to Fig. 41, let Z_a equal the angular change at A due to the moment, $X_a = -1$, when all joints except A are considered fixed. Angular changes, Z_b, Z_e, etc., are similarly defined. For any value of X_a other than —1, the angular change at A is equal to $Z_a\,X_a$. For Joint A, Fig. 41:

$$M_{AB} + M_{AD} + M_{AC} = \frac{X'_a}{E} = X_a \qquad (58)$$

Therefore,

$$K_{AB}\,\beta_{AB}\,\theta_A + K_{AD}\,\beta_{AD}\,\theta_A + K_{AC}\,\beta_{AC}\,\theta_A = X_a \qquad (59)$$

When $X_a = -1$,

$$Z_a = \frac{-1}{K_{AB}\,\beta_{AB} + K_{AD}\,\beta_{AD} + K_{AC}\,\beta_{AC}} \qquad (60)$$

The denominator in Equation (60) is the moment at A due to $\theta_A = 1$.

In general:

$$Z_a = \frac{-1}{\Sigma\,K_{A(\;)}\,\beta_{A(\;)}} \qquad (61)$$

in which, the summation is over all the members meeting at Joint A. In a like manner, Z_b, Z_e, etc., are computed.

The Determination of X_{ba}, X_{ab}, X_{be}, X_{eb}, Etc.—The moment at B, due to $\theta_A = 1$, is equal to X_{ba}. All other angles are considered zero. For any other value of θ_A than 1, the moment at $B = \theta_A\,X_{ba}$. The moments, X_{ab}, X_{be}, etc., are similarly defined; thus,

$$X_{ba} = X_{ab} = K_{AB}\,\beta'_{BA} = K_{AB}\,\beta'_{AB} \qquad (62)$$

From the quantities, Z_a, Z_b, etc., and X_{ab}, X_{be}, etc., the angles, θ', may be determined for any external load when the effect of the deflection, d, is neglected.

In using this method for fixed loads, one calculates the fixed-end moments at each joint due to the loads and applies to each joint the algebraic sum of the fixed-end moments at the joint as an external load with proper sign.

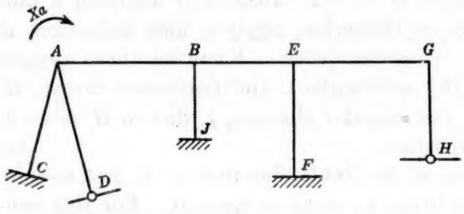

FIG. 41

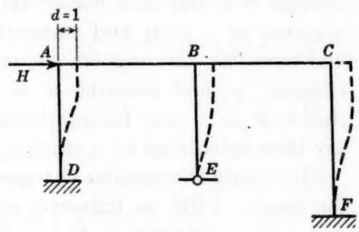

FIG. 42

If there are more than two load conditions to be considered, it is quicker to compute influence values for the moments. In the calculations for influence values, apply $X_a = -1$ (Fig. 41) as the only load on the structure, and compute the corresponding angular changes. Compute angular changes for $X_b = -1$, $X_e = -1$, and $X_g = -1$, respectively. Calculate the moments at the ends of the members for these conditions of loading. The actual moments are in direct proportion to the values of X_a, X_b, X_e, and X_g, due to the actual load. In computing influence lines, the following procedure gives the full solution very quickly. First, compute the angular changes for $X = -1$ for the joint farthest to the left (or right). For instance, in Fig. 41, compute the angular changes for $X_a = -1$. Then, apply $X_b = -1$ as the only load and compute the first approximation, $\Delta\,\theta'_b$. It would be necessary to perform the same operations on this value of $\Delta\,\theta'_b$ that were performed on $\Delta\,\theta'_b$ in the previous computation for $X_a = -1$. Therefore, the angular turns for $X_b = -1$ are equal to the angular turns for $X_a = -1$ multiplied by $\Delta\,\theta'_b$ due to $X_b = -1$, divided by $\Delta\,\theta'_b$ due to $X_a = -1$. In computing θ'_a, only the sum of the corrections from the first computations should be used.

Then, proceed to Joint E and apply Set $X_e = -1$. From Maxwell's theorem, the angular change at E due to a unit moment at A is equal to the angular change at A due to a unit moment at E; likewise, for B and E. Then, the values of θ'_A and θ'_B may be written at once for $X_e = -1$. Since this θ'_B is the final correct value of θ'_B, it is necessary only to make corrections for the joints to the right of B. Compute the angular turn at E due to $X_e = -1$. Compute the angular turn at E due to θ'_B and add the results, which equal $\Delta\,\theta'_e$; calculate $\Delta\,\theta'_g$; and make corrections only to those two angles. Finally, proceed to Joint G.

It is not necessary that the calculations for the load conditions $X_b = -1$; $X_e = -1$; and $X_g = -1$ be carried through as outlined herein, but it is believed that this method of computation requires the least amount of work.

The Effect of Horizontal Movement.—In the preceding discussion, the angles, θ', were obtained for the condition that the frame had no horizontal deflection. It is proposed now to calculate this deflection and corrections to the angles, θ', due to the deflection. The moments acting at the top and bottom of the columns in a frame produce a horizontal thrust, or shear in the

columns which causes a deflection, d, and a change in the slope of the tangent to the elastic curve of the members. From Maxwell's law, a moment, —1, at B (see Fig. 42) produces a deflection, d, which is equal to the angular change at B due to a horizontal force, $H = -1$. Instead of applying a unit moment at a joint and computing the deflection, apply a unit deflection, d, and calculate the angular changes, α, at the joints. Knowing these angular changes, α, and knowing $d = 1$ (by assumption), the horizontal thrust, H, due to $d = 1$, may be computed. The angular changes, ρ, due to $H = -1$, are then calculated by a simple proportion.

To obtain the angular changes, α, at the joints, due to $d = 1$, first assume the beam, ABC, as infinitely stiff; then, $\theta_A = \theta_B = \theta_C = 0$. For this condition, the moments at the top of the column will be, for $d = 1$ (see Fig. 42):

$$M_{AD} = -\frac{K_{AD}\,\beta''_{AD}}{h_{AD}};\quad M_{BE} = -\frac{K_{BE}\,\beta_{BE}}{h_{BE}};\ \text{and}\ M_{CF} = -\frac{K_{CF}\,\beta''_{CF}}{h_{CF}}.$$

From which,

$$X_{a(d)} = -\frac{K_{AD}\,\beta''_{AD}}{h_{AD}} \qquad (63)$$

$$X_{b(d)} = -\frac{K_{BE}\,\beta_{BE}}{h_{BE}} \qquad (64)$$

$$X_{c(d)} = -\frac{K_{CF}\,\beta''_{CF}}{h_{CF}} \qquad (65)$$

The moments, $X_{a(d)}$, $X_{b(d)}$, and $X_{c(d)}$, are considered as external loads applied at their respective joints. The calculations for the angular changes, α, may be made exactly as in the case of neglecting horizontal deflection, previously described. If the computations for the θ' values have been carried through for influence lines, the more direct way is to proceed, as follows: Since the angular turns have already been found, due to a moment, —1, at each joint, the values of α may be determined by considering that $X_{a(d)}$, $X_{b(d)}$, and $X_{c(d)}$ act one at a time, and adding the effects. Multiply the moment considered acting, by the values of θ' previously obtained when the corresponding joint was loaded with $X = -1$, assuming no horizontal deflection. Repeat this process for all joints and add algebraically. It is obvious that these sums are the angular changes, α, due to $d = 1$. Note that the angles, θ', were calculated for $X = -1$, and the moments about the joints due to $d = 1$ are also negative.

The horizontal shear $-H$, in the columns is the sum of the moments at the top and bottom of the columns divided by their respective heights. Since $-H$ is the horizontal shear due to $d = 1$, the angular changes, ρ, due to $H = -1$, equals $\dfrac{\alpha}{H}$.

Computation of H.—It is best to separate the computations for H into two parts: First, the effect, H', produced by the angular changes, α; and, second, the effect, H'', produced by the deflection, $d = 1$.

For a member restrained at both ends:

$$H' = \frac{K_{AB}\,[\beta_{AB}\ \alpha_A + \beta'_{AB}\ \alpha_B]}{h_{AB}} + \frac{K_{AB}\,[\beta_{BA}\ \alpha_B + \beta'_{BA}\ \alpha_A]}{h_{AB}}$$

$$= \frac{K_{AB}\,\beta''_{AB}\ \alpha_A}{h_{AB}} + \frac{K_{AB}\,\beta''_{BA}\ \alpha_B}{h_{AB}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (66)$$

$$H'' = -\ \frac{K_{AB}\,\beta''_{AB}}{h^2_{AB}} - \frac{K_{AB}\,\beta''_{BA}}{h^2_{AB}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (67)$$

and,

$$H = H' + H'' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (68)$$

For a single-story frame, such as that shown in Fig. 42:

$$H' = -\,X_{a(d)}\ \alpha_A - X_{b(d)}\ \alpha_B - X_{c(d)}\ \alpha_C \ldots\ldots\ldots\ldots\ldots (69)$$

in which,

$$-X_{a(d)} = \frac{K_{AD}\,\beta_{AD} + K_{AD}\,\beta'_{DA}}{h_{AD}} = \frac{K_{AD}\,\beta''_{AD}}{h_{AD}} \ldots\ldots\ldots (70)$$

$$-X_{b(d)} = \frac{K_{BE}\,\beta_{BE}}{h_{BE}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (71)$$

$$-X_{c(d)} = \frac{K_{CF}\,\beta_{CF} + K_{CF}\,\beta'_{FC}}{h_{CF}} = \frac{K_{CF}\,\beta''_{CF}}{h_{CF}} \ldots\ldots\ldots (72)$$

and,

$$H'' = +\,\frac{X_{a(d)}}{h_{AD}} + \frac{X_{b(d)}}{h_{BE}} + \frac{X_{c(d)}}{h_{CF}} + \frac{X_{d(d)}}{h_{AD}} + \frac{X_{f(d)}}{h_{CF}} \ldots\ldots\ldots (73)$$

The values of ρ may next be calculated by solving the formula:

$$\rho = \frac{\alpha}{-H} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (74)$$

that is, $\rho_A = \dfrac{\alpha_A}{-H}$, $\rho_B = \dfrac{\alpha_B}{-H}$, $\rho_C = \dfrac{\alpha_C}{-H}$, etc.

According to the sign system adopted, it will be seen that a negative H or a negative moment around a joint produces a positive deflection; therefore, for $X = -1$, $d = \rho$. For a horizontal load of -1 at the top of columns, $d = \dfrac{1}{-H}$.

The final values of the angular turns are obtained by adding the effects produced by the assumption that d equals zero, and the angular turns produced by considering the effect of d acting alone. These final angles may be written:

$$\theta = \theta' + \alpha\, d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

For instance, for the case of $X_a = -1$: $\theta_A = \theta'_A + \alpha_A\ d_A$; $\theta_B = \theta'_B + \alpha_B\ d_A$; etc. The factor, d_A, signifies the deflection due to $X_a = -1$. The angles are now completely solved and the moments may be calculated by substitution into general slope-deflection formulas. The method may be applied to multi-storied frames, secondary stresses, and other problems involving elastic deformations.

Example No. 1.—The writer has chosen the example solved by Professor Cross as an illustration of the method of making the calculations for a fixed condition of loading when the effect of horizontal movement is neglected.

(1).—The fixed-end moments in all the members are first calculated. Since Joint F is considered hinged, the moment at C in CF is computed from the relation: $H_{CF} = C_{CF} + \frac{C_{FC}}{2} = +80 + \frac{60}{2} = +110$, and the moment at $F = 0$.

The values of the external moments, X_b, X_c, etc., acting on the joints are next obtained by adding algebraically the fixed-end moments at the joints. In this connection it is to be noted that the external moment considered as acting around a joint is of the same sign as the fixed-end moment considered as acting on the loaded member. For example, $X_c = C_{CB} + H_{CF} + C_{CD} + C_{CG} = +100 + 110 - 200 + 50 = +60$. Similarly, $X_b = -100$; $X_d = +100$; and, $X_e = C_{ED} +$ cantilever moment $= 0 - 10 = -10$.

(2).—The angular changes at each joint due to a moment, -1, acting at the joint, while all other angles are considered zero, are next computed; thus $Z_c = \frac{-1}{4\,(4 + 5 + 1) + 3 \times 2} = -0.02174$. Similarly, $Z_b = -0.04167$; $Z_d = -0.03125$; and $Z_e = -0.08333$.

(3).—The moment at the fixed end of a member due to a unit rotation at the other end is given by the quantities, X_{cb}, X_{dc}, etc. For example, $X_{cb} = X_{bc} = 2 \times 4 = 8$; $X_{dc} = X_{cd} = 2 \times 5 = 10$; and $X_{ed} = X_{de} = 2 \times 3 = 6$.

(4).—The product, $X_{cb}\,Z_c$, therefore, gives the angular turn at C due to a unit rotation of Joint B. These quantities are computed for carrying over effects of angular turns from right to left, and from left to right. Table 7 gives these values for the author's Fig. 1. All the constants required for the determination of the angular turns have been found, and the computations for the actual rotations may be made.

TABLE 7.—COMPUTATION OF ANGLE CHANGES AT JOINTS INDICATED, CAUSED BY A UNIT ROTATION AT ADJACENT JOINTS

Joint	FOR CARRYING ANGLES TO THE RIGHT		FOR CARRYING ANGLES TO THE LEFT	
	Product $X\,Z$	Value	Product $X\,Z$	Value
B	....	....	$X_{bc}\,Z_b$	-0.3333
C	$X_{cb}\,Z_c$	-0.1739	$X_{cd}\,Z_c$	-0.2174
D	$X_{dc}\,Z_d$	-0.3125	$X_{de}\,Z_d$	-0.1875
E	$X_{ed}\,Z_e$	-0.5	....	....

For the first approximations, $\Delta\,\theta'$, the computations are as follows: At Joint B there is an external moment of -100; hence, $\Delta\,\theta'_B = -100 \times Z_b = -100 \times -0.04167 = +4.167$, which is the value of θ'_B for the moment, -100, at Joint B, when all other joints are held fixed.

At Joint C there is an external moment of $+60$, hence, $\Delta\,\theta'_C = +60 \times Z_c + (\Delta\,\theta'_B)\,(X_{cb}\,Z_c) = +60 \times -0.02174 + 4.167 \times -0.1739 = -2.029$,

which is the value of θ'_C for the moment, $+ 60$, at Joint C, when all other joints are held fixed, plus the effect of the angular turn ($\Delta\theta'_B = 4.167$) at Joint B.

The first approximations of θ'_D and θ'_E, computed exactly in the same manner as those for θ'_B, are found to be: $\Delta\,\theta'_D = -\,2.491$, and $\Delta\,\theta'_E = +\,2.078$, respectively (see Column (2), Table 8).

TABLE 8.—TOTAL ANGLE CHANGES, EXAMPLE NO. 1

Joint	FIRST APPROXIMATION	SUCCESSIVE CORRECTIONS					TOTALS	
	$\Delta\,\theta'$	θ'_1	θ'_2	θ'_3	θ'_4	θ'_5	θ' after two corrections	θ' after five corrections
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
B....	+ 4.1667	+ 0.4675	....	+ 0.01096	....	− 0.00087	+ 4.635	+ 4.644
C....	− 2.029	+ 0.626	− 0.0813	+ 0.0484	− 0.0019	+ 0.00452	− 1.484	− 1.433
D....	− 2.491	− 0.3898	− 0.1703	− 0.0525	− 0.0145	− 0.00628	− 3.051	− 3.124
E....	+ 2.0788	....	+ 0.2800	....	+ 0.0335	....	+ 2.358	+ 2.392

Corrections are made to these approximate quantities to obtain more exact values. The first corrections are made from the joint, E, to the left.

The first correction for the angle, D, is made by computing the angular change at D due to the angle, $\Delta\,\theta'_E = +\,2.078$; thus, $\theta'_{1D} = \Delta\,\theta'_E\,X_{de}\,Z_d = +\,2.078 \times -\,0.1875 = -\,0.3898$ (Table 8, Column (3)).

The first correction for the angle, C, is made by computing the angular change at C, due to $\Delta\,\theta'_D + \theta'_{1D}$; thus, $\theta'_{1C} = (\Delta\,\theta'_D + \theta'_{1D})\,X_{cd}\,Z_c = -2.879 \times -\,0.2174 = +\,0.626$.

All other corrections are made in the same manner. It is to be noted that they are made only on increments of the angles which have not been previously carried over to the adjacent joint. The total angular change is obtained for any joint by adding the first approximation and the corrections to the angle.

The simplicity of the arrangement of computations shown in Table 8 makes chance of error or failure to consider any term almost negligible. For comparison of results, the moments have been computed for the angles obtained after the second correction and after the fifth correction. The results which are given in Table 9 indicate that for all practical purposes the computations might properly have been stopped with the second correction.

TABLE 9.—COMPARISON OF MOMENTS AFTER TWO AND FIVE CORRECTIONS, RESPECTIVELY

Number of corrections	COMPARISON OF BENDING MOMENTS										
	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CF}	M_{CD}	M_{CG}	M_{GC}	M_{DC}	M_{DE}	M_{ED}
5.......	18.576	+37.152	−37.160	+114.224	+101.402	−259.90	+44.264	+22.132	+23.19	−23.136	+ 9.96
2.......	+18.54	+37.08	−37.71	+114.34	+101.10	−260.92	+44.06	+22.03	+24.14	−22.26	+10.40

Example No. 2.—The real simplicity and value of this method of computation is brought out in the calculations for influence lines for moving loads. The calculations are first carried through for the angles considering that the frame has no horizontal deflection. Then, these angles are corrected for the effect of the deflection. The computations are made for the frame shown in Fig. 43. An external moment, $X = -1$, is considered as the only load on the frame and the angular changes are computed for this condition of loading. The applied moment, $X = -1$, is assumed to act first at Joint A and then at each of the other joints in succession.

The quantities, Z_a, Z_b, etc., and X_{ba}, X_{cb}, etc., and the products, $X_{ba}\ Z_b$, $X_{cb}\ Z_c$, etc., are computed exactly as explained in Example No. 1, and are arranged conveniently in Table 10.

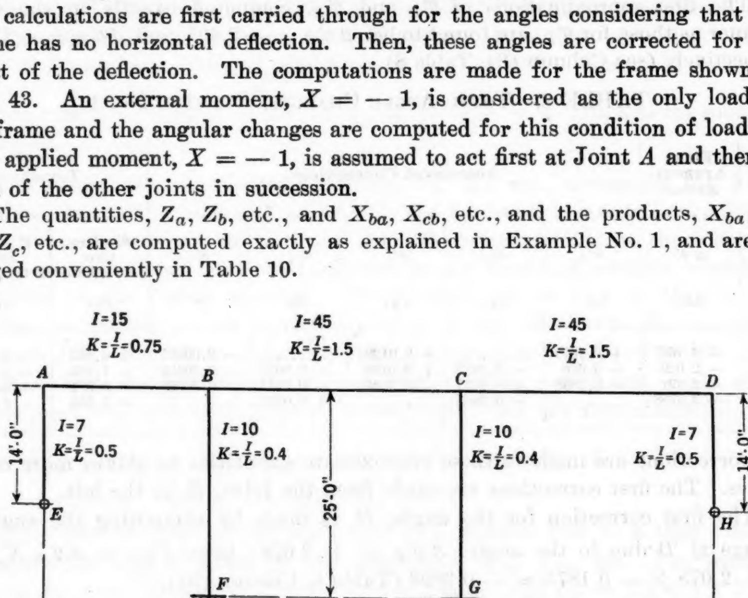

FIG. 43

The computations of the angular turns for $X_a = -1$, etc., are given in Table 11. Explanations of some of the items will serve to make this table clear:

(1).—At A the only load is the moment, -1; hence, for $\Delta\ \theta'_A$, $\Delta\ \theta'_A = -1 \times Z = +\ 0.222$.

(2).—There is no external load at B, and the rotation is caused by the effect of $\Delta\ \theta'_A$; therefore, for $\Delta\ \theta'_B$, $\Delta\ \theta'_B = \Delta\ \theta'_B\ X_{ba}\ Z_b = +\ 0.222 \times -0.1415 = -\ 0.0314$.

(3).—Likewise, the rotation at C is caused by the effect of $\Delta\ \theta'_B$, or $\Delta\ \theta'_C = \Delta\ \theta'_B\ X_{cb}\ Z_c = -\ 0.0314 \times -\ 0.221 = +\ 0.00694$.

(4).—For $\Delta\ \theta'_D$, $\Delta\ \theta'_D = \Delta\ \theta'_C\ X_{dc}\ Z_d = +\ 0.00694 \times -\ 0.4 = -\ 0.00278$.

Corrections to these approximate values are made as in Example No. 1.

Computations of the angular turns for $X_b = -1$ are shown in Table 11(b). The value of θ'_A was obtained directly by Maxwell's law. The final values of θ'_B, θ'_C, and θ'_D, may be computed from the relation:

$$\theta' = \frac{\Delta\ \theta'_B\ (\text{for } X_b = -1)}{\Delta\ \theta'_B\ (\text{for } X_a = -1)} \times \theta'\ (\text{for } X_a = -1)$$

that is, the angular turns, θ'_B, θ'_C, θ'_D, for $X_b = -1$, are $\left(\frac{+\ 0.0944}{-\ 0.0314} = \right)$ 3.0 times as large as for $X_a = -1$.

Computations of the angular turns, θ'_A and θ'_B, when $X_c = -1$ (see Table 11(*c*)), may be written directly by Maxwell's law. Since they are the final values they require no correction, and computations must be made for θ'_C and θ'_D only, thus, $\Delta\,\theta'_C = -1 \times Z_c + \theta'_B\, X_{cb}\, Z_c = -1 \times -0.0735 - 0.0256 \times -0.221 = +0.07916$, and, $\Delta\,\theta'_D = \Delta\,\theta'_C\, X_{dc}\, Z_a = +0.07916 \times -0.4 = -0.03170$. Corrections are made only to these two angles, and are carried through in the usual manner.

TABLE 10.—COMPUTATION OF ANGLE CHANGES, EXAMPLE NO. 2, CAUSED BY A UNIT ROTATION AT ADJACENT JOINTS

Joint	EXTERNAL MOMENTS, X		ANGLE CHANGE, Z, for $M = -1$	
	Symbol	Computation	Symbol	Computation
A	X_{ab}	$2 \times 0.75 = 1.5$	Z_a	$\dfrac{-1}{3 \times 0.5 + 4 \times 0.75} = -0.222$
B	X_{bc} X_{ba}	$2 \times 1.5 = 3.0$ $2 \times 0.75 = 1.5$	Z_b	$\dfrac{-1}{4\,(0.75 + 1.5 + 0.4)} = -0.0944$
C	$X_{cd} = X_{cb}$	$2 \times 1.5 = 3.0$	Z_c	$\dfrac{-1}{4\,(1.5 + 1.5 + 0.4)} = -0.735$
D	X_{dc}	$2 \times 1.5 = 3.0$	Z_d	$\dfrac{-1}{4 \times 1.5 + 3 \times 0.5} = -0.1333$

ANGLE CHANGES AT THE JOINTS INDICATED, CAUSED BY A UNIT ROTATION AT ADJACENT JOINTS

Joint	FOR CARRYING ANGLES TO THE RIGHT		FOR CARRYING ANGLES TO THE LEFT	
	Product XZ	Value	Product XZ	Value
A	....	....	$X_{ab}\, Z_a$	−0.333
B	$X_{ba}\, Z_b$	−0.1415	$X_{bc}\, Z_b$	−0.283
C	$X_{cb}\, Z_c$	−0.221	$X_{cd}\, Z_c$	−0.221
D	$X_{dc}\, Z_d$	−0.4	....	....

Computations of the angular turns, θ'_A, θ'_B, and θ'_C, for $X_d = -1$, may be written directly (see Table 11(*d*)), as follows: $\theta'_D = -1 \times Z_d + \theta'_c\, X_{dc}\, Z_d = -1 \times -0.1333 - 0.03475 \times -0.4 = +0.1472$. Table 11 gives the values of the rotations when the horizontal deflection is neglected.

To correct for the effect of horizontal deflection a unit deflection is assumed at the top of the columns, and the moments at the top and bottom of the columns are computed on the assumption that the beam, $A\,B\,C\,D$, is infinitely stiff; thus, $X_{a(d)} = X_{d(d)} = \dfrac{-3 \times 0.5}{14} = -0.1071$, and $X_{b(d)} = X_{c(d)} = X_{f(d)} = X_{g(d)} = \dfrac{-6 \times 0.4}{25} = -0.0961$.

The moments at the top of the column are considered as external moments on the joints. They are assumed to act one at a time, and the results are added for the total effect. The effect of $X_{a(d)}$ is obtained by multiplying the angles found for $X_a = -1$ by the value of $X_{a(d)}$; and likewise for all other moments, as shown in Table 12.

TABLE 11.—COMPUTATION OF ANGLE CHANGES, EXAMPLE NO. 2

	(a)' For the Load, $X_a = -1$					(b) For the Load, $X_b = -1$	
θ'	$\Delta\theta'$	θ'_1	θ'_2	θ'_3	Total θ'	$\Delta\theta'$	Total θ'
θ'_A	$-1 \times Z_a = +0.222$	$+0.01118$	....	$+0.000615$	$+0.23379$	....	-0.03539
θ'_B	-0.0314	-0.00214	-0.00158	-0.000268	-0.03589	$-1 \times Z_b = +0.0944$	$-3 \times -0.03539 = +0.10650$
θ'_C	$+0.00694$	$+0.000614$	$+0.000821$	$+0.000127$	$+0.00850$	....	$-3 \times 0.0085 = -0.0256$
θ'_D	-0.00278	....	-0.000574	-0.000074	-0.003428	....	$-3 \times -0.003428 = +0.01032$

$$\frac{+0.0944}{-0.0314} = -3.0$$

	(c) For the Load, $X_c = -1$					(d) For the Load $X_d = -1$	
θ'	$\Delta\theta'$	θ'_1	θ'_2	θ'_3	Total θ'	$\Delta\theta'$	Total θ'
θ'_A	....	....	....	....	$+0.00850$	....	-0.003428
θ'_B	....	....	....	....	-0.0256	....	$+0.010320$
θ'_C	$-1\, Z_c = +0.07350$ $+0.00566$ $\overline{+0.07916}$	$+0.0070$	....	$+0.000618$	$+0.08678$	....	-0.03475
θ'_D	-0.03170	....	-0.0028	-0.00025	-0.03475	$-1\, Z_d +0.1333$ $= +0.0189$ $\overline{+0.1472}$	$+0.14720$

$-0.0256 \times -0.221 = +0.00566$ $\qquad$ $-0.03475 \times -0.4 = +0.0189$

TABLE 12.—COMPUTATION OF ANGULAR TURN, α

	A	B	C	D
$X_{a(d)}\ \theta'$ (for $X_a = -1$)	$+0.0250$	-0.0038	$+0.00091$	-0.000368
$X_{b(d)}\ \theta'$ (for $X_b = -1$)	-0.0034	$+0.0102$	-0.00246	$+0.000994$
$X_{c(d)}\ \theta'$ (for $X_c = -1$)	$+0.000816$	-0.00246	$+0.00833$	-0.00334
$X_{d(d)}\ \theta'$ (for $X_d = -1$)	-0.000368	$+0.001108$	-0.00372	$+0.0158$
α	$+0.022048$	$+0.005048$	$+0.003060$	$+0.013086$
$\rho = \frac{\alpha}{-H}$	$+0.843$	$+0.1929$	$+0.1170$	$+0.500$

TABLE 13.—COMPUTATION OF HORIZONTAL REACTION, H

	H'		H''
$-X_{a(d)}\,\alpha_A$	$+0.00236$	$X_{a(d)} \div h_{AE}$	-0.00766
$-X_{b(d)}\,\alpha_B$	$+0.000485$	$2\,X_{b(d)} \div h_{BF}$	-0.00769
$-X_{c(d)}\,\alpha_C$	$+0.000294$	$2\,X_{c(d)} \div h_{CG}$	-0.00769
$-X_{d(d)}\,\alpha_D$	$+0.00140$	$X_{d(d)} \div h_{DH}$	-0.00766
Total..........	$+0.004539$	Total........	-0.03070

$$H = H' + H'' = -0.026161$$

TABLE 14.—FINAL ANGULAR TURNS AND DEFLECTIONS FOR VARIOUS LOAD CONDITIONS

	LOAD CONDITIONS				
	$X_a = -1$	$X_b = -1$	$X_c = -1$	$X_d = -1$	$H = -1$
	$+0.2338$ $+0.0186$	-0.0354 $+0.0042$	$+0.0085$ $+0.0026$	-0.0034 $+0.0110$	
θ_A	$+0.2524$	-0.0312	$+0.0111$	$+0.0076$	$+0.8430$
	-0.0354 $+0.0042$	$+0.1065$ $+0.0010$	-0.0256 $+0.0006$	$+0.0103$ $+0.0025$	
θ_B	-0.0312	$+0.1075$	-0.0250	$+0.0128$	$+0.1929$
	$+0.0085$ $+0.0026$	-0.0256 $+0.0006$	$+0.0868$ $+0.0004$	-0.0347 $+0.0015$	
θ_C	$+0.0111$	-0.0250	$+0.0872$	-0.0332	$+0.1170$
	-0.0034 $+0.0110$	$+0.0103$ $+0.0025$	-0.0347 $+0.0015$	$+0.1472$ $+0.0065$	
θ_D	$+0.0076$	$+0.0128$	-0.0332	$+0.1537$	$+0.5000$
d	$+0.8430$	$+0.1929$	$+0.1170$	$+0.5000$	$+38.225$

Computations for H are as follows:

$$H = H' + H''$$

$$H' = -X_{a(d)}\,\alpha_a - X_{b(d)}\,\alpha_b - X_{c(d)}\,\alpha_c - X_{d(d)}\,\alpha_d$$

and,

$$H'' = \frac{X_{a(d)}}{h_{AE}} + \frac{X_{b(d)} + X_{f(d)}}{h_{BF}} + \frac{X_{c(d)} + X_{g(d)}}{h_{CG}} + \frac{X_{d(d)}}{h_{DH}}$$

The operations indicated are listed in Table 13. The angles, ρ, are computed by the formula, $\rho = \frac{\alpha}{-H}$, as follows:

$$\rho_A = \frac{\alpha_A}{-H} = \frac{+0.02205}{-(-0.02616)} = +0.843$$

$$\rho_B = \frac{\alpha_B}{-H} = \frac{+0.005048}{-(-0.02616)} = +0.1929$$

$$\rho_C = \frac{\alpha_C}{-H} = \frac{+0.00306}{-(-0.02616)} = +0.1170$$

and,

$$\rho_D = \frac{\alpha_D}{-H} = \frac{+0.013086}{-(-0.02616)} = +0.500$$

The values, ρ, in addition to being the angular turns at the joints due to $H = -1$, are also the deflections at the top of the columns due to $X = -1$ at the various joints; for instance: for $X_a = -1$, $d_A = \rho_A$; for $X_b = -1$, $d_B = \rho_B$; etc.

The angular turns at the joints, including the effect of the horizontal deflection, are computed from the equation: $\theta' = \theta' + \alpha\, d$. For instance, for $X_a = -1$: $\theta_A = \theta'_A + \alpha_A\, d_A$; $\theta_B = \theta'_B + \alpha_B\, d_A$; etc.

The final angular turns and deflections for the various load conditions are shown in Table 14.

The moments are computed by substitution into the general slope-deflection equations. From these values the effect of any load on the frame may be evaluated.

E. C. Hartmann,[20] Jun. Am. Soc. C. E. (by letter).[20a]—The author's method of analyzing continuous frames by distributing fixed-end moments is so simple and direct that there seems to be small reason for suggesting a variation of the system outlined in his paper. However, the writer would like to point out a slightly different system for distributing the fixed-end moments which sometimes may prove useful where the separate effects of a number of loading conditions are to be studied. If it serves no other purpose, this discussion will at least emphasize the flexibility of the Cross method.

The system that the writer has in mind can probably be best illustrated by reference to the case of an ordinary continuous girder. Assume a four-span structure with the points of support lettered, successively, from left to right, A, B, C, D, and E; and assume that it is desired to study the moments

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[20] Research Engr., Aluminum Research Laboratories, Aluminum Co. of America, New Kensington, Pa.

[20a] Received by the Secretary, March 24, 1931.

caused by a number of different loading conditions. The solution suggested is as follows:

1.—Assume any hypothetical unbalanced negative moment on the left side of Support *B*. Distribute this single unbalanced moment in the manner described in the paper, obtaining the resulting moments at each point of support.

2.—Apply the same hypothetical unbalanced moment at the right side of Support *B*. The resulting moments at each point of support can be determined in this case by inspection from the values obtained in Step 1.

3.—Repeat Steps 1 and 2 for Supports *C* and *D*.

4.—Determine the fixed-end moments for one of the loading conditions to be studied, say, Condition I.

5.—Take one of these fixed-end moments and determine the resulting moment at each point of support by proportion from the values obtained in Step 1, Step 2, or Step 3.

6.—Repeat Step 5 for each of the fixed-end moments for loading Condition I. The final moments at the supports for that loading condition may now be obtained by adding algebraically the results obtained for the several fixed-end moments.

7.—The moments at the supports resulting from other loading conditions may be obtained by repeating Steps 4, 5, and 6.

It is important to note in this example, that, regardless of how many loading conditions are investigated, the number of distributions of unbalanced moments never exceeds three. Having the results of these three distributions on hand, one can tabulate the remainder of the work for easy slide-rule calculation.

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AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

DISCUSSIONS

HIGHWAYS AS ELEMENTS OF TRANSPORTATION

Discussion

BY MESSRS. EDWIN F. WENDT, H. W. HUDSON, AND FRED LAVIS

EDWIN F. WENDT,[1] M. AM. SOC. C. E. (by letter).[1a]—This paper is a notable contribution to the science of engineering in its relation to transportation. When the late Arthur M. Wellington, M. Am. Soc. C. E., wrote his book entitled "Economic Theory of Railway Location" fifty years ago, the Engineering Profession did not foresee that in 1930 the same principles would be applied to highway design by reason of the growth of traffic due to the extensive use of motor cars.

The author calls attention to the fact that, until recent times, highways were the developments of old trails made by man before the era of wheeled vehicles. The Hon. Henry W. Temple, Representative in Congress of the 25th Pennsylvania District, elaborated this same fact in an address delivered before the Service Clubs of Washington, Pa., September 25, 1930, on the subject "Some Historical Associations of the National Pike". This address contains much interesting information for the engineer.

Mr. Lavis very properly points out that engineering and economics are indissolubly linked together, and this applies to highways as well as railways. He has demonstrated by his thorough analysis of the problem of highway design that the true engineer is an economist, and that economics is essentially a part of engineering. The rise, the development, and the phenomenal growth in the number of motor cars require the design and construction of highways in accordance with scientific principles so that there will be a true economic proportion between costs of such construction and the cost of operation of the motor vehicles which use the highways. The author is a pioneer in the application of the principles of the economic theory of railway location to the design, construction, and operation of modern highways.

H. W. HUDSON,[2] M. AM. SOC. C. E. (by letter).[2a]—The author has referred to a change made in the alignment and profile of Route 25 between the Penn-

NOTE.—Author's closure. The paper by Fred Lavis, M. Am. Soc. C. E., was published in August, 1930, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: October, 1930, by Messrs. Walter Loring Webb, W. W. Crosby, and Jay Downer; and December, 1930, by Messrs. S. Johannesson, D. P. Krynine, Harold M. Lewis, and Donald M. Baker.

[1] Cons. Engr., Washington, D. C.

[1a] Received by the Secretary, December 30, 1930.

[2] Engr. in Chg. of Constr., State Highway Comm., Jersey City, N. J.

[2a] Received by the Secretary, March 2, 1931.

sylvania Railroad crossing in Jersey City and a point west of the Passaic River in Newark, which change was made subsequent to his connection with the project.

The permit issued by the War Department for the construction of the movable bridges having 35-ft. normal clearance which had been adopted for the crossings of the Hackensack and Passaic Rivers, stipulated that the existing bascule bridge crossing the Hackensack River on Lincoln Highway should be removed following the completion of Route 25. The permit also stipulated that the Passaic River Bridge should be raised to provide a minimum clearance of 35 ft.

Realizing the impracticability of closing Lincoln Highway, due to the removal of the Hackensack River Bridge, as called for by the War Department's permit, the Highway Commission decided to construct a tunnel to take the place of the bridge. This decision led to considerable adverse criticism by interests more or less directly affected. It was appreciated, however, that the cost of this tunnel, as well as the reconstruction of the Passaic River Bridge, required by the War Department permit, would add so materially to the total expenditure, that, under these conditions, the adopted plan, involving movable bridges, might not remain the most economical. Further studies, therefore, were made of a proposal involving tunnels under both rivers and an open cut below the meadow level across the Kearny Peninsula, as well as a plan involving bridges with sufficient under-clearance to permit the use of fixed spans.

Either of these proposals would not affect the river traffic, and it was assumed, therefore, that the War Department would not require any changes in the Lincoln Highway bridges in connection with issuing a permit for either of these types of river crossings, and hence no consideration was given to this contingency in preparing the comparative estimates. In the former, it was necessary to consider the possibility of ultimately having to construct a roof over this cut coincident with the development of the contiguous property. In such event, it was realized that the problem of ventilation would be a serious element.

The alignment for both the tunnel and high-level viaduct plans was identical and was found to be several hundred feet shorter than that of the adopted location which contemplated movable bridges with a normal clearance of 35 ft. Either proposal involved a saving in curvature, but increased the rise and fall over the 35-ft. clearance scheme.

The construction and right-of-way costs of the route involving movable bridges (including the raising and reconstruction of the Passaic River Bridge and the construction of tunnels under the Hackensack River, both on Lincoln Highway), was found to be greater than the high-level viaduct route. Similarly, it was found that the construction costs of tunnels under the rivers, with the open depressed cut through the meadows, would be in excess of either the 35-ft. plan, or the high-level viaduct. Investigation of automobile operating costs of the three proposals also indicated the high-level viaduct project to be the least costly.

The Highway Commission, therefore, rescinded its action as regards the 35-ft. level bridge location and adopted the 135-ft. high-level viaduct. The latter will provide a highway entirely free from any interruptions to its traffic caused by water-borne traffic.

Another change made subsequent to the author's connection with the work was that of the three crossings of the Elizabeth River, in Elizabeth, N. J. As the waterway at this point is under the jurisdiction of the War Department, the permit for the crossings called for the construction of movable bridges. Authority was given to erect these structures as fixed spans until such time as it might be necessary to provide for navigation, when the operating mechanism would have to be installed.

The construction through Elizabeth provided for all streets to intersect or cross the highway at grade. It was evident that a serious traffic hazard would obtain at the crossing of Elizabeth Avenue which is the principal east and west thoroughfare. It was finally arranged to carry the highway over this street on a bridge, and to continue it on an elevated structure southward to, and including, the three river crossings where fixed span bridges with a vertical clearance 22 ft. above mean high water were used. The War Department waived the requirement of movable bridges and approved of the construction of the fixed-span bridges with the clearance mentioned. The wisdom of this viaduct construction has been fully justified.

FRED LAVIS,[3] M. AM. SOC. C. E. (by letter).[3a]—The writer appreciates the generally favorable comment on his paper, especially because of the prominence of many of the commentators in the field of modern highway construction along the lines of development which take into due consideration the effects of such highways and the details of their location on the economic operation of the vehicles using them.

The commentators recognize also the nice balance which must be preserved between the bare application of any economic theory in a problem of this kind and the "other" factors which affect the location and design. It is the ability to strike this balance accurately, giving proper weight to economic as well as technical considerations, which marks the true engineer.

The discussion by Mr. Lewis is particularly interesting as he gives a complete outline of the various types of construction which have been developed thus far in the endeavor of engineers to find solutions of these problems to fit the needs of different localities and conditions.

Mr. Downer also refers only too briefly to the development of the highways of the Westchester County Park System. This project originated primarily as a park development along the Bronx River Valley between the northerly end of New York City and Kensico Reservoir and was the practical realization of a dream for the preservation of scenic beauty, with a highway as an incidental feature. Before the Bronx River Parkway was completed, however, the highway feature had grown to be a most important factor, and fortunately it was realized by Mr. Downer at an early stage in the development that special attention would have to be paid to its design as an "element in transportation".

[3] Pres., International Rys. of Central America, New York, N. Y.

[3a] Received by the Secretary, March 31, 1931.

In the later developments through the other three valleys running northerly from New York City the landscape features of the parkways have been developed, as adjuncts of the highways, and the design of these highways has been more and more influenced in these later years by traffic requirements. Mr. Downer and the Westchester County Park Commission deserve the highest congratulations for having achieved that equitable balance between traffic needs, and the artistic and other requirements, which is so eminently the earmark of true engineering.

It will be realized, however, that the location of Route 25, in New Jersey, through an entirely urban and industrial and manufacturing district, did not permit this type of development and this all serves to point out the need of the individual study of each problem by properly qualified engineers in order to find the solution best adapted to the needs of each particular case.

Perhaps a word may be permitted in regard to highway widths. On Route 25, New Jersey, the elevated west side highway in New York, and others of this type, no provision is made for other than moving traffic, in which case the writer believes an odd number of lanes (five of 10 ft. each in the case of Route 25) is indicated. The principal reasons for this are pointed out in the discussion by Mr. Baker.

There seems, however, to be a general tendency to establish State Highway widths for main routes at 20 ft., or 40 ft. (two or four lanes). With evenly balanced traffic, and the need of giving some consideration to parking requirements, this may be correct; but the traffic in the vicinity of big cities, especially big terminal cities like New York, is not balanced and as Mr. Baker points out, the extra lane is justified. This is also more particularly the case when part of the traffic is commercial and is especially noticeable on some roads leading to New York on Sunday afternoons and evenings, with many vehicles carrying produce for the markets, at the time that the peak load of tourist traffic is returning to the city.

In regard to Route 25, New Jersey, it was estimated that four full lanes of moving traffic would be required to take care of peak loads, and that it was only ordinary prudence to provide an extra lane for emergencies and passing to keep four full lanes continuously moving.

The discussion by Colonel Hudson, outlining the changes in design on two sections of Route 25, is a welcome contribution and serves to complete the record of the project. Referring to the map and profile, Figs. 1 and 2, the section first referred to is that between Stations 180 and 330. For the four years (1925 to 1929) during which this matter was studied by W. G. Sloan, M. Am. Soc. C. E., State Highway Engineer, and the writer, there were long drawn out controversies with the municipal authorities and others as to the relative merits of a line on this general location, with crossings of the Hackensack and Passaic Rivers by tunnels or with movable bridges with 40-ft. clear head-room. The writer and Major Sloan were opposed to a tunnel line with the connecting low-level trenches for this distance of nearly 3 miles.

The writer is entirely in accord with the final decision to build this section of the highway as a viaduct with 135-ft. clear head-room over the rivers

so long as the War Department and the New Jersey River and Harbor Board insisted on this as the minimum clearance for fixed spans. It may be pointed out, however, that if such a structure had been proposed in the first place, it probably would have imperiled the entire project, partly because of the expense involved, but largely because of the general impression which would have been created in the minds of the public that it was too grandiose.

In passing also it may be noted that, in the writer's opinion, neither the War Department nor the Board of Commerce and Navigation of New Jersey has an adequate appreciation of the relative or economic value of terrestrial as compared with the water-borne traffic of this area, and, consequently, the tax-payers of the State are unduly penalized, not only in expenditures for unnecessarily costly structures, but in the costs of traffic delays.

In regard to the change at Elizabeth, the project as finally carried out, except for a slightly higher elevation over the Elizabeth River, is almost exactly that recommended by the writer and adopted by the Highway Commission early in 1926. It was strenuously opposed, however, at that time, by almost all the interests in Elizabeth. Many and various proposals for other routes were studied and suggested during the next three years. Agreements were reached only to be thrown over at the last moment and Colonel Hudson is entitled to a great deal of credit for finally persuading "the powers that be" to agree to the project as finally constructed, which the writer understands is now considered a most acceptable solution of the problem.

The original report of the Advisory Board of the Highway Commission, of which Major Sloan was Chairman, showed an estimated cost of approximately \$20 000 000 for the entire 13 miles from the Holland Tunnel to Elizabeth, with bridges over the Hackensack and Passaic Rivers (for tunnels, \$28 000 000), and the writer has always felt that the Commission was entitled to the very highest commendation for the splendid manner in which it upheld the report of its engineer and decided to go ahead with a project of this magnitude with little—if any—precedent to guide it.

After the work was finally authorized, more detailed studies showed necessary or desirable developments which brought the estimated cost of the project, as finally developed under the writer's direction, up to a total of approximately \$35 000 000; and the final cost, with the change to the high level as referred to in Colonel Hudson's discussion, to approximately \$40 000 000.

To the layman this may appear to be another instance of the inadequacy of engineers' estimates, but consideration should be given to two most important factors. First, the very great number of different interests which had to be served or satisfied. Above all, there was the State, then the counties, the municipalities, the chambers of commerce, the railroads, the public utilities, the navigable waterways interests, individual manufacturing and commercial interests, and, finally, the private citizens, all of which had a voice in determining some detail of the project. The very important change in Newark shown in Fig. 7, which added more than \$1 000 000 to the cost of the highway, is a good illustration. Second, this was a pioneer project of its kind. Very few people were able to visualize it or what the plans of the Highway Com-

mission contemplated. In the writer's opinion this perhaps was fortunate, because if its magnitude had been realized it is doubtful whether it would have been carried out.

Finally, the writer feels that not only is Major Sloan entitled to the highest credit for the original conception of the project as the only reasonable solution of the problem of highway transportation through this area, but in even greater degree for his patience and persistence in insisting that it be carried through on the basis of true engineering principles.

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DISCUSSIONS

STRESSES IN GRAVITY DAMS BY PRINCIPLE OF LEAST WORK

Discussion

BY MESSRS. FERNAND CAMPUS, A. FLORIS, ROBERT E. GLOVER, AND P. WILHELM WERNER

FERNAND CAMPUS,[28] Esq. (by letter).[28a]—Since the paper is purely theoretical, the writer will first consider the question from that point of view. He believes that the author does not make correct use of the principle of least work. This principle gives rise to great difficulties in interpretation and application. Föppl expressly warns against the errors it may induce.[29] The author gives a faulty interpretation of Föppl's text in the final reference in Appendix II.

The theorem of least work is only applicable in the case of elasticity. This point is duly established in the works of Clapeyron, Lamé, Castigliano, Menabrea, Boussinesq, Maurice Lévy, Bertrand de Fontviolant,[30] Föppl,[31] and others. The principle of least work and the principle of virtual velocities are not, as the author seems to consider them, essentially different; they are identical or, more correctly, they are only slightly different expressions for one and the same principle or assumption—the hypothesis of elasticity. This is demonstrated clearly by de Fontviolant and especially by Föppl's treatise.[32] Föppl states the principle of virtual velocities in the form of the law of least work of deformation, and he proves beyond any possible doubt that these two principles are only valid when the hypothesis of elasticity is assumed.

If so, the stresses must satisfy the author's Equation (43)[33] which has not the same meaning as Föppl's compatibility equations;[34] or Airy's stress function may be determined, which must satisfy Equation (42). The author improperly

NOTE.—The paper by B. F. Jakobsen, M. Am. Soc. C. E., was published in September, 1930, *Proceedings*. Discussion of the paper has appeared in *Proceedings*, as follows: September, 1930, by the late William Cain, M. Am. Soc. C. E.; November, 1930, by Messrs. William Gore, L. J. Mensch, Johannes Skytte, and Lars R. Jorgensen; January, 1931, by Messrs. C. A. P. Turner, Donald P. Barnes, and Anthony Hoadley; February, 1931, by Messrs. Fred. W. Ely, and M. H. Gerry, Jr.; and April, 1931, by Messrs. Fredrik Vogt and Eugene Kalman.

[28] Prof., Liége Univ., Liége, Belgium.

[28a] Received by the Secretary, January 24, 1931.

[29] "Drang und Zwang," 1920, Article 9, p. 68.

[30] "Les méthodes modernes de la resistance des matériaux," Paris, 1920.

[31] "Drang und Zwang," 1920, Article 13, p. 99.

[32] *Loc. cit.*, Article 9, pp. 61–66, inclusive.

[33] "Résistance des matériaux et elasticité," G. Pigeaud, p. 736, Paris, 1920.

[34] "Drang und Zwang," 1920, Articles 7 and 40.

identifies this formula with the compatibility equation in plane elasticity. As a matter of fact, Equations (42) and (43) are identical, but quite different from the compatibility equations.

In the last statement of Appendix II, the author has misinterpreted Föppl's text. Föppl states[35] explicitly that the stresses, n_x, n_y, n_z, t_{xy}, t_{yz}, and t_{zx}, satisfy the hypothesis of elasticity, thus, also, the compatibility equations, and he mentions very small variations, $\delta\, n_x$, $\delta\, n_y$, $\delta\, n_z$, $\delta\, t_{xy}$, etc., which do not satisfy them, but satisfy the equilibrium equations. He proves then that the corresponding value of $\delta\, L$ is zero; therefore, the work done by the stresses, n_x, n_y, n_z, t_{xy}, etc. is a minimum. This relates only to the stresses satisfying the elasticity equations, and not to the other internal forces. It is true[36] that the stresses computed by the Ritz method do not satisfy the elasticity equations; but this method is only approximate; it cannot exactly satisfy the laws of elasticity and all the formulas and principles derived from these laws.

In other words, the basic principle is that of elasticity. The elasticity equations, the author's Equation (43), the Airy function (the author's Equation (42)), the principle of virtual velocities, the principle of least work (which states that the total differential of the work is zero and not certain partial differentials as in the Ritz method)—all these formulas and principles are but slightly different forms for the expression of one and the same fundamental hypothesis—elasticity. The principle of the superposition of effects depends also entirely and exclusively upon this fundamental hypothesis. This principle is necessarily true in this case, when elasticity is assumed to apply. The writer will now prove that the author did not properly consider these fundamental facts in his paper.

Arbitrarily, Mr. Jakobsen assumes the law,

$$n_y = A_y + B_x + C\,\frac{x^2}{y} + F\,\frac{x^3}{y^2}$$

which is homogeneous. In order to determine A, B, C, and F, he uses the equations of static equilibrium and to these he joins, by means of the Ritz method, the relations, $\frac{dL}{dC} = 0$ and $\frac{dL}{dF} = 0$.

There is nothing to justify him in considering this the principle of least work; it is only an approximation commonly designated as the Ritz method. The author arrives at formulas for n_y, n_x, and t, and he ascertains that these quantities do not satisfy the compatibility equations, which means exactly that they do not harmonize with the hypothesis of elasticity. As a matter of fact, they should not, because they were obtained by the Ritz method. Consequently, the formulas and principles derived from elasticity are not applicable to these stresses, although the author assumes that elastic deformations occur. For instance, the work of 16 653 000 ft-lb. in Table 3, is not a minimum. The minimum is 17 218 000 ft-lb., as computed by means of the linear solution. If the work calculated, using the author's stresses, is 3.26% less than the minimum, this is due entirely to the inaccuracy of the computation. Equations

[35] "Drang und Zwang," 1920, Article 13.
[36] *Loc. cit.*, 1920, Article 55, p. 325.

(14), (29), (30), and (31), for the work, only have a bearing upon stresses that satisfy the assumption of elasticity. The internal forces as given by Mr. Jakobsen are very far from satisfying this condition. This also explains the noticeable differences between the stresses given in Articles VII and VIII.

At the down-stream face, $n_y = -76\,704$ (adding values in Articles VII and VIII), instead of $-80\,909$ from Article V (difference, 5.35%). At the up-stream face, $+41\,715$ is computed instead of 37 650 (Article V). Here, the difference is 10.8 per cent. It is very simple to explain the differences. The principle of superposition is not applicable to the stresses computed with the author's equations; they do not satisfy the law of elasticity. The differences are not small, so this seems to the writer an objection that all engineers will appreciate.

Now, examine the differences between the principle of elasticity and the author's formulas. It can be proved by accepting the author's assumptions, that the exact solution is linear (ignoring all lateral deformations, in accordance with Appendix III, which is applicable to every condition).

The writer has pointed out that the law chosen by the author (Equation (15)) is homogeneous. In the case of a straight line, $x = ky$, in the interior of the dam:

$$n_y\, k = y\,(A + Bk + Ck^2 + Fk^3) = Ky \ldots\ldots\ldots\ldots (92)$$

The validity of this statement is not limited to the up-stream face as given by the author in Article VI; it is general. Furthermore, $n_x\, k = K'\, y$ and $t\, k = K''\, y$. This variety of stress distribution is designated homothetic, that is, similar and similarly placed.

The summit of the triangular section of the dam is the homothetic center. In cases of homothetic stress distribution it is easy to prove that the solution satisfying the hypothesis of elasticity and all its consequences—for example, least work—is the linear solution. A very simple demonstration is given in the work by Pigeaud.[37]

When the author assumes his homogeneous law, he implicitly assumes a homothetic condition. In this case the exact solution satisfying elasticity is the linear law; and this is the only solution that verifies the elasticity equations and the formula of work, and the principles of least work and of superposition. The stresses found by the author by means of the Ritz method show great divergence from the exact solution. This demonstrates the degree of error that the Ritz method may involve. It is sufficiently illustrated by Figs. 4, 5, 6, and 7, of the paper. This explains also all the aforementioned differences; the author assumes elasticity, but he does not respect this assumption.

In Article II, Mr. Jakobsen seems to consider the assumption of linear stress distribution as arbitrary; and, in Article III, he maintains that it is equivalent to stating that horizontal planes remain plane. It is possible that this opinion is based on a memoir by Lévy[38] which is now becoming obsolete. Incidentally, the author mentions the consideration of normal instead of horizontal sections, thus referring to the work of Résal.

[37] "Resistance des matériaux et elasticité," Paris, 1920, pp. 736 and 737.

[38] Comptes rendus de l'Academie des Sciences, Paris, 1895.

These points of view have been displaced by a more precise conception of the linear law. Consider a triangular dam of infinite height, infinite resistance, and infinite elasticity; the stress distribution is a plane and a homothetic one. Each unit of volume weighs exactly as much as any other; the pressure is hydrostatic and the water level is at exactly the same height as the ridge of the dam. Under these conditions the general equations of internal equilibrium and of elasticity lead to a linear stress distribution which is identical to that of Lévy. The demonstration of this theorem is to be found in the works of many authors. The writer needs only to mention Pigeaud's treatise[39], in which the starting point is the equation,

$$\frac{d^2 (n_x + n_y)}{dx^2} + \frac{d^2 (n_x + n_y)}{dy^2} = 0 \ldots\ldots\ldots\ldots (93)$$

Mr. S.-D. Carothers, of London, England, has given a new and very elegant demonstration of this theorem. He has applied the Airy function and polar co-ordinates in accordance with Love's "Treatise on Elasticity" (4th Edition).[40]

It should be noted that this theorem does not imply any assumption as to the deformations or the conservation of plane horizontal sections. On the contrary, the writer has proved,[41] in a most general way, that assuming the linear stress distribution, plane horizontal sections actually become curved and are changed into cylinders with conic sections.

Therefore, the correct solution for the equations of stress, corresponding to the author's assumptions (elasticity and homogeneity, implying an homothetic condition), is the linear solution, which satisfies the principles of least work and of superposition of the effects of forces. The question of whether to consider horizontal or normal sections is unimportant; they do not influence the equations. The writer does not mean to say that the linear solution is consistent with actual conditions; but neither does the correct solution depend on the principle of least work. It depends, as the author states in the final sentence of Article III, upon the influence of the foundation. All authors agree on this point, Résal, Mesnager, Pigeaud, Haegelen, etc.[42]

The linear solution is based on the assumption of an infinitely large dam in order to avoid the difficulties resulting from the foundation. There is only one inaccuracy in the linear solution, namely, dams are actually finite and they have a foundation. Therefore, to the linear stress distribution, must be superposed the secondary stresses arising from the contact between the base of the dam and the foundation soil. A mathematical solution for this problem is impossible with the present limitations of science. It is clear, however, that these stresses are most important near the base, and that at a certain distance above, the linear law is only slightly altered. This is demonstrated in photo-elastic tests by Mesnager, according to Haegelen's opinions.[43]

[39] "Resistance des matériaux et elasticité," Paris, 1920.

[40] *Proceedings*, First International Cong. for Concrete and Reinforced Concrete, Liége, 1930.

[41] *Bulletin* de la Classe des Sciences de l'Academie Royale de Belgique; and *Le Genie Civil*, January 3, 1931.

[42] See, also, F. Campus, "Conditions de stabilité des barrages à gravité en béton," Paris et Liége, pp. 37 *et seq.*

[43] Congrès de l'Union Internationale des producteurs et distributeurs d'énergie electrique, Paris, 1928.

The writer believes, furthermore, that this is proved (within the limits of experimental errors) by Fig. 9, referring to experiments by Ottley and Brightmore. In this diagram, however, no allowance is made for conditions at the two extremities of Section 6-6. It indicates continuity between the dam and the base, representing the soil and producing singular stresses in the angles. Nothing of that kind exists in a real foundation. Sections 5-5, 4-4, and 3-3 (Fig. 9) are less influenced by these unreal conditions, and the writer's remarks are more plausible for such places, although conditions at these sections are influenced by the fact that the dam is not triangular and also by corresponding stresses originating at the crest (especially in Section 3-3). Mathematical proof for linear stress is only possible for strictly triangular dams. The author mentions (not without reason) the numerous facts which may influence the reliability of the test results. The approximations made in reading the instruments may be added to the list. Hence, extreme caution is necessary in interpreting tests relating to dams. The interpretation based on the English investigation in favor of the linear law is quite reasonable.

In any case a non-linear stress distribution based on the theory of elasticity and taking into account the complications resulting from the foundations, in order to have a more approximate image of the real stress law in a dam, should not have as its starting point a homogenous formula implying homothetic stress distribution. The foundation suppresses homothetic distribution; this is undoubtedly the point which the author ignored.

Briefly, Mr. Jakobsen's paper is instructive, not only because it represents a meritorious effort, but also because of the objections that arise in discussing it. Theoretically, the method contains a radical error; practically, the writer does not think it is superior to the ordinary method and he believes it will meet with no success. It is more complicated even for the case of a vertical up-stream face which is the only one discussed by the author. In an actual dam this face is inclined; but this would have occasioned great analytical difficulties. Hence, the author's method is not as general as the linear law.

The writer agrees with Mr. Jakobsen that it is less important to limit the maximum compression at the down-stream face than to require minimum compression at the up-stream face when the reservoir is filled. However, the writer believes that the formulas presented by Lévy are satisfactory for application in this case. The advantage of imposing more severe conditions, in accordance with the author's remarks in Articles IX and X, seems quite illusive considering that many dam constructors are so severely opposed to the section proposed by Maurice Lévy.

The writer has been somewhat loath to discuss this paper, especially since he was forced to criticize the contribution of an author whose valuable work in the study of dams he holds in great esteem.

A. FLORIS,[44] ESQ. (by letter).[44a]—The author questions the correctness of basing the analysis of gravity dams on a linear distribution of the vertical normal stress, and proposes a new law of stress distribution that differs widely

[44] Civ. Engr., Los Angeles, Calif.

[44a] Received by the Secretary, February 11, 1931.

from that determined by the rule of the trapezoid. His investigations are interesting because they lead to results that are not in accord with the findings of other writers. The possibility of disclosing the reason for the discrepancy between the classical theory and that presented by the author is very attractive to any one of an inquiring turn of mind. On the other hand, to those who are concerned merely with the application of the theory to practical cases, such differences are bewildering. For this reason, any discussion on this able paper must necessarily attempt to reconcile these important differences.

Mr. Jakobsen assumes a triangular profile of unlimited height acted upon by the water pressure and the weight of the dam; that is, he neglects the influence of the foundation on the stress distribution in the dam. Exactly the same assumptions are made in the classical theory, developed by Maurice Lévy,[45] Paul Fillunger,[46] E. Selényi,[47] and others. It follows, therefore, that although the problem is the same in both cases, the results are quite different. The writer will explain the reason for this difference in some detail.

Fig. 34 illustrates a dam with a triangular profile. It has a vertical up-stream face and unlimited height, and a thickness equal to unity. The water surface is assumed to be level with the top of the dam which is also the origin of the co-ordinates, x and y. The equations for the up-stream and down-stream boundaries are, respectively, $x = 0$ and $x = \tan \alpha\, y = \kappa\, y$. Furthermore, assume that the stress distribution in the interior follows the linear law as expressed by the formulas,

$$\sigma_x = a\,x + b\,y \qquad (94)$$

$$\sigma_y = c\,x + f\,y \qquad (95)$$

and,

$$\tau_{xy} = e\,x + h\,y \qquad (96)$$

Consider a rectangular element, as shown in Fig. 34, in the interior of the dam. From the condition that the horizontal and vertical forces acting on this element must be in equilibrium, the following equations may be written:

$$\frac{\partial\, \sigma_x}{\partial_x} + \frac{\partial\, \tau_{xy}}{\partial_y} = 0 \qquad (97)$$

$$\frac{\partial\, \sigma_y}{\partial_y} + \frac{\partial\, \tau_{xy}}{\partial_x} + \gamma_c = 0 \qquad (98)$$

The boundary condition will be expressed by the equations:

$$\sigma_{n1} = \sigma_x = -\gamma_w\, y \qquad (99)$$

$$\sigma_{n2}\ (x, \kappa\, x) = 0 \qquad (100)$$

$$\tau_{n1} = \tau_{xy} = 0 \qquad (101)$$

and,

$$\tau_{n2}\ (x, \kappa\, x) = 0 \qquad (102)$$

[45] "Sur l'équilibre élastique d'un barrage en maçonnerie à section triangulaire," par Maurice Lévy, *Comptes rendus* des Séances de l'Académie des Sciences de Paris, Vol. 127, 1898, p. 10.

[46] "Drei wichtige Spannungszustände des keilförmigen Körpers," von Dr.-Ing. Paul Fillunger, *Zeitschrift für Mathematik und Physik*, Vol. 60, 1912, p. 273.

[47] "Sul regime elastico nelle dighe di tipo gravità," by E. Kalman, *L'Energia Elettrica*, February, 1927.

In these equations, γ_c and γ_w are the specific weights of the dam and of the water, respectively. If the stresses, σ_x, σ_y, and τ_{xy}, in two mutually perpendicular planes are known, then the stresses in a plane forming an angle, α, with the vertical will be given by the following equations:

$$\sigma_n = \sigma_x \cos^2 \alpha + \sigma_y \sin^2 \alpha - 2 \tau_{xy} \cos \alpha \sin \alpha \ldots\ldots\ldots\ldots (103)$$

and,

$$\tau_n = (\sigma_x - \sigma_y) \sin \alpha \cos \alpha + \tau_{xy} (\cos^2 \alpha - \sin^2 \alpha) \ldots\ldots\ldots (104)$$

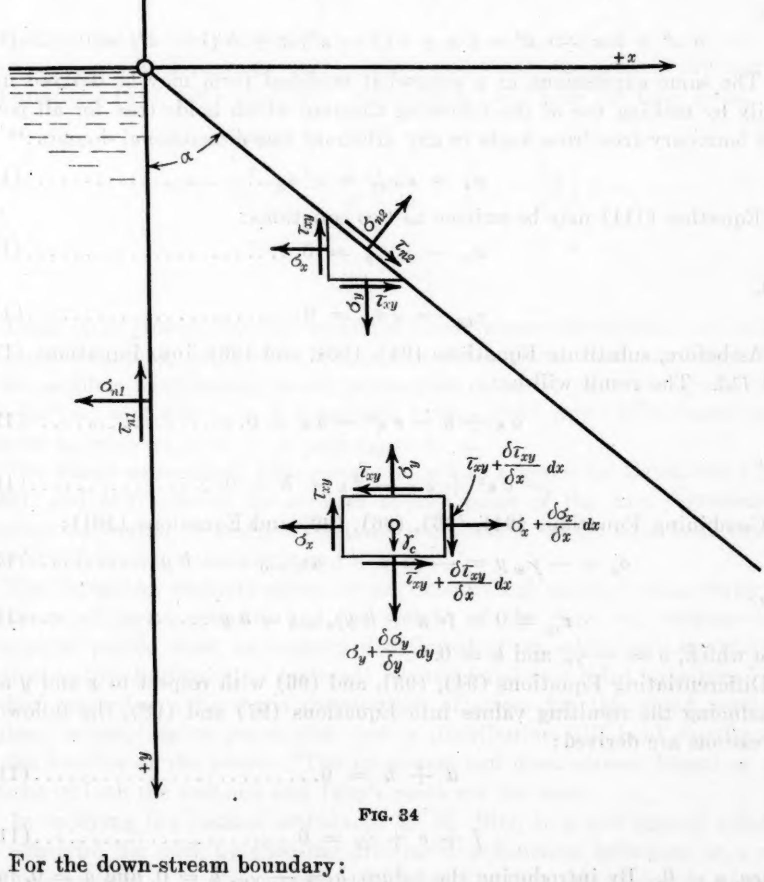

FIG. 34

For the down-stream boundary:

$$\sin \alpha = \frac{\kappa}{\sqrt{1 + \kappa^2}} \ldots\ldots\ldots\ldots\ldots\ldots (105)$$

and,

$$\cos \alpha = \frac{1}{\sqrt{1 + \kappa^2}} \ldots\ldots\ldots\ldots\ldots\ldots (106)$$

in which, $\kappa = \tan \alpha$, as before.

Introducing in Equations (103) and (104) the values of Equations (94), (95), (96), (105), and (106), Equations (107) and (108) are obtained, for the down-stream boundary, as follows:

$$\sigma_{n2} = [a\,\kappa + b + c\,\kappa^3 + f\,\kappa^2 - 2\,e\,\kappa^2 - 2\,h\,\kappa]\,y = 0 \quad \ldots\ldots (107)$$

and,

$$\tau_{n2} = [a\,\kappa^2 + b\,\kappa - c\,\kappa^2 - f\,\kappa + e\,(1 - \kappa^2)\,\kappa + h\,(1 - \kappa^2)]\,y = 0. \quad (108)$$

Because these expressions are valid for all values of y, it follows that:

$$a\,\kappa + b + c\,\kappa^3 + f\,\kappa^2 - 2\,e\,\kappa^2 - 2\,h\,\kappa = 0 \quad \ldots\ldots\ldots\ldots (109)$$

and,

$$a\,\kappa^2 + b\,\kappa - c\,\kappa^2 - f\,\kappa + e\,(1 - \kappa^2)\,\kappa + h\,(1 - \kappa^2) = 0 \quad \ldots\ldots (110)$$

The same expressions in a somewhat modified form may be derived more easily by making use of the following theorem which holds true for all points of a boundary free from loads in any arbitrary two-dimensional domain:[48]

$$\sigma_x = \kappa\,\tau_{xy} = \kappa^2\,\sigma_y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (111)$$

Equation (111) may be written as two equations:

$$\sigma_x - \kappa\,\tau_{xy} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (112)$$

and,

$$\tau_{xy} - \kappa\,\sigma_y = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (113)$$

As before, substitute Equations (94), (95), and (96), into Equations (112) and 113. The result will be:

$$a\,\kappa + b - e\,\kappa^2 - h\,\kappa = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (114)$$

and,

$$-\,c\,\kappa^2 + e\,\kappa - f\,\kappa + h = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (115)$$

Combining Equations (94), (95), (96), (99) and Equations (101):

$$\sigma_x = -\,\gamma_w\,y = -(a\,\kappa + b\,y)_{\kappa=0} = -\,b\,y \quad \ldots\ldots\ldots\ldots (116)$$

and,

$$\tau_{xy} = 0 = (e\,\kappa + h\,y)_{\kappa=0} = h\,y \quad \ldots\ldots\ldots\ldots\ldots\ldots (117)$$

from which, $b = -\,\gamma_w$ and $h = 0$.

Differentiating Equations (94), (95), and (96) with respect to x and y and introducing the resulting values into Equations (97) and (98), the following expressions are derived:

$$a + h = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (118)$$

and,

$$f + e + \gamma_c = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (119)$$

Hence, $a = 0$. By introducing the values, $b = -\,\gamma_w$, $h = 0$, and $a = 0$, into Equations (109) and (110):

$$-\,\gamma_w + c\,\kappa^3 + f\,\kappa^2 - 2\,e\,\kappa^2 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (120)$$

and,

$$-\,\gamma_w\,\kappa^2 - c\,\kappa^3 - f\,\kappa^2 + \kappa\,x^2\,(1 - \kappa^2) = 0 \quad \ldots\ldots\ldots\ldots (121)$$

[48] "Sulla validità dell' potesi Navier nelle mensole," by E. Kalman, Reale Instituto Lombardo di Scienze e Lettere, Estratto dai Rendiconti, Vol. LXI, Fasc. XVI–XX, 1928.

Solving for e:

$$e = -\frac{\gamma_w}{\kappa^2} \qquad (122)$$

Substituting this value of e in Equation (119),

$$f = -\left(\gamma_c - \frac{\gamma_w}{\kappa^2}\right) \qquad (123)$$

Introducing Equations (122) and (123) into Equation (120):

$$c = \left(\gamma_c - 2\,\frac{\gamma_w}{\kappa^2}\right)\frac{1}{\kappa} \qquad (124)$$

Making use of the parameter, $b = -\gamma_w$, $h = 0$, and $a = 0$, as well as those expressed by Equations (122), (123), and (124), the general expressions in Equations (94), (95), and (96), respectively, will take the final form,

$$\sigma_x = -\gamma_w\, y \qquad (125)$$

$$\sigma_y = \left(\gamma_c - 2\,\frac{\gamma_w}{\kappa^2}\right)\frac{x}{\kappa} - \left(\gamma_c - \frac{\gamma_w}{\kappa^2}\right) y \qquad (126)$$

and,

$$\tau_{xy} = -\frac{\gamma_w}{\kappa^2}\, x \qquad (127)$$

Thus, it is proved that the tentative linear stress distribution, as defined by Equations (94), (95), and (96), can be fitted to the boundary conditions of the problem by choosing its six parameters properly. For the special case, $\alpha = 45°$, $\gamma_w = 1$ and $\gamma_c = 2$, Equations (125), (126), and (127), respectively, become $\sigma_x = -y$; $\sigma_y = -y$; and $\tau_{xy} = -x$.

The linear expressions with respect to x and y given by Equations (125), (126), and (127) define the stresses at any point of the dam regardless of whether horizontal or inclined sections are used.[49] These equations express Lévy's well-known stress regimen.

The foregoing analysis shows in an unequivocal manner that, under the influence of water pressure and the weight of the dam, the stresses in a triangular profile must be linearly distributed if an additional straight-line section (either horizontal or inclined) is considered as a third boundary. The author prescribes a parabolic distribution of stress for this third boundary without attempting to prove that such a distribution will hold equally true in the interior of the profile. The up-stream and down-stream boundary conditions in both the author's and Lévy's cases are the same.

In applying the method introduced by W. Ritz, it is not merely sufficient to minimize the work by choosing arbitrarily a function belonging to a competitive system; it is also necessary to take care that this system is correctly chosen from the infinite number of possible choices, any one of which might supply a minimum function by a proper choice of the parameters. For example, among n competitive systems, there are n solutions which satisfy the condition of least work. In all problems of this nature it is necessary to give the characteristics of the functions that are permitted to compete. For

[49] "Ueber den Spannungszustand in Staumauern", von E. Kalman, *Beton und Eisen*, May 5, 1930, p. 174.

this purpose the shape of the curve that represents these functions is assumed only approximately and upon this choice the correct solution of the problem depends to a large extent. Otherwise, in order to obtain the correct solution, it would be necessary to take into consideration all competitive systems that contain a sufficient number of functions in series, each series having a sufficient number of terms.

For the reasons explained in the foregoing remarks, the writer is forced to conclude that the results of the author's theory are, unfortunately, misleading. The Ritz method of least work has been applied successfully to the design of circular tanks[50] and arch dams,[51] but the writer considers that stress analysis of triangular gravity dam profiles of unlimited height is not possible by this method.

To prove the correctness of his theory, the author refers to the well-known English test on models. It is questionable whether the analysis of these two divergent cases can be made to agree. The stress distribution in the models was influenced considerably by the foundation conditions at the dam. This influence was disregarded entirely by Mr. Jakobsen. In the upper parts of the models the stresses seem to follow a linear rather than a parabolic law. This is in agreement with theory because, according to Saint-Venant's principle, the stresses in these models cannot be influenced materially at elevations considerably above the base. The stresses will follow the linear law quite closely, so that Lévy's regimen will be approximately correct at such elevations.

The writer doubts very much whether progress can be made in the analysis of a triangular dam profile when the influence of the foundation upon the dam is neglected. The important problem that still remains to be solved is that of considering the dam and its unlimited rock foundation acting together as an elastic body. This is impossible at present, however, due to the great mathematical difficulties involved. Therefore, it is necessary to make certain assumptions regarding the deformation of the rock or to be guided by the results of experiment.

As is well known, the stress components in any two-dimensional domain can be expressed by means of the Airy stress function satisfying a partial differential equation of the fourth order under given boundary conditions. (See the author's Equations (40) and (42).) Among the particular solutions of this differential equation are also those that can be expressed in the form of polynomials.

Professor Karl Wolf has expressed the normal loads (water pressure) at the up-stream boundary of a triangular dam profile by an arbitrary polynomial of the nth degree with respect to y.[52] It is evident in this case that the tangential

[50] "Die Berechnung der Spannungen in zylindrischen Behälterwänden mit veränderlichem Querschnitt," von Theodor Pöschl, *Sitzungsberichte* der Kaiserl. Akademie der Wissenschaften in Wien, June, 1912. See, also, "Berechnung von Behältern nach neueren analytischen und graphischen Methoden," von Dr. Th. Pöschl und Dr. K. von Terzaghi, Berlin, 1926, p. 127.

[51] "Sulle piastre curve", Nata dell'ing. Prof. Kalman, Estratto dai Rendiconti del R. Instituto, Lombardo di scienze e lettere, Adunanza dell 11 Aprile, 1929, Serie II, Vol. LXII, Fasc. VI–X.

[52] "Zur Integration der Gleichung $\Delta\,\Delta\, F = 0$ durch Polynome im Falle des Staumauernproblems", von Karl Wolf, *Sitzungsberichte* der Kaiserl. Akademie der Wissenschaften in Wien, February, 1914.

loads are zero. On the other hand, the down-stream boundary of the dam is free from loads while the rigid foundation represents the third boundary of the triangular profile.

The coefficients can be determined in such a way that the polynomial will express with sufficient accuracy not only the water pressure within the limits, $y = 0$ (at the top) and $y = h$ (at the bottom), but, also (because of the assumed rigidity of the foundation), the vertical displacements for $x = h$ shall be a minimum. This can be accomplished in the following manner. The height and the base of the profile are each divided into n equal parts. For each of the n intervals of the height of the profile, the integral of the polynomial is made approximately equal to the corresponding water pressure (by the method of least squares). For each n interval of the base ($y = h$), the integral of the vertical displacements is made equal to zero, in order to satisfy the condition imposed by assuming that the base is rigid. This procedure supplies n linear equations for the determination of n unknown coefficients in the assumed polynomial.

Consider a polynomial of the fifth degree with respect to y, in which the down-stream face of the dam makes an angle of 45° with the up-stream face, and the assumed height of the profile as well as the specific weight of water are assumed equal to unity. Let the specific weight of the dam equal 2. For this case, Professor Wolf determines the following expression for the horizontal normal stress at the up-stream face:

$$(\sigma_x)_{x=0} = -y + 0.116\,y - 1.101\,y^2 + 3.325\,y^3 - 4.000\,y^4 + 1.668\,y^5 \quad (128)$$

At the base of the dam, that is, for $y = h$, Equation (128) gives a value for σ_x which differs from the water pressure by only 7 per cent. The first term of this equation corresponds to the stress given by Equation (125) for the special case, namely, $\sigma_x = -y$.

The remaining stresses at the base and at the mid-height of the dam are, respectively,

$$(\sigma_y)_{y=1} = -0.836 - 0.939\,x + 0.829\,x^2 + 0.904\,x^3 - 2.452\,x^4 + 1.766\,x^5 \quad (129)$$

$$(\tau_{xy})_{y=1} = -1.288\,x + 1.677\,x^2 + 3.495\,x^3 + 1.898\,x^4 + 0.490\,x^5 \quad (130)$$

and,

$$(\tau_{xy})_{y=0.5} = -1.123\,x + 0.321\,x + 0.459\,x^3 - 1.057\,x^4 + 0.490\,x^5 \quad (131)$$

Equation (129) corresponds to the rule of the trapezoid in Lévy's regimen. In this special case of the stress distribution the trapezoid becomes a rectangle. For different values of x in Equation (129), corresponding values of σ_y are, as follows:

x	$(\sigma_y)_{y=1}$
0	0.836
0.1	0.930
0.3	1.035
0.5	1.084
0.7	1.080
0.9	0.903
1.0	0.730

When plotted, these values appear in the form of a parabolic curve concave upward. At the up-stream or down-stream faces the stress is smaller than that resulting from Equation (126) ($\sigma_y = -y$), while at the middle of the section the stress is somewhat greater. In Equation (129) y does not appear because the expression is valid only for $y = 1$. For $x = 0$, in Equation (129), $\sigma_y = -0.836$, while for $y = 1$ in the special case of Equation (126), $\sigma_x = 1$. It is important to note that the shearing stress toward the top of the dam rapidly approaches that determined by Lévy's linear law. Even at the base the distribution of the shearing stress does not differ radically from that given by Lévy's regimen. For example, in both cases, the shearing stresses vanish at the up-stream face (for $x = 0$) throughout the height of the dam.

It is true that Professor Wolf's solution may be open to certain objections. However, it represents an elastic state which is correct while that of Mr. Jakobsen is not. Furthermore, the results obtained by this method are not extreme so that they cannot be rejected as useless. Unfortunately, the same reasoning cannot be applied to the author's findings. Before accepting a new theory in which there are substantial differences when compared with methods in current use, it is necessary that such theory be proved to be correct beyond doubt.

Further advance can be made in the theory of stress distribution in triangular gravity dams by modifying Lévy's stress regimen at the base to take care of the conditions actually existing at that place.

In analyzing the stress condition in triangular dam profiles near the foundation, Professor Kalman has proved that the water pressure at the bottom of the reservoir has an appreciable influence on the stresses in the dam itself.[53] This is a fact that has been overlooked in the English tests and in all theoretical investigations on stresses in dams. Furthermore, Professor Kalman shows by a rigorous analysis that the stresses at the bottom of a triangular profile cannot be linear. By cutting the dam just above its foundation and applying Lévy's stresses at the cut surfaces as loads, Professor Kalman finds that both surfaces do not remain congruent after deformation; and this fact proves conclusively that the stresses in that part of the dam cannot be linear. If they were linear, the separated surfaces must remain congruent after deformation. In reality, of course, the dam is not cut at this place, and both surfaces must remain congruent before and after deformation. Therefore, the stresses developed therein must necessarily differ from those given by Lévy's regimen.

Basing his thesis on the theory that the two separated surfaces must be congruent, Professor Kalman investigates the application of corrective stresses on these surfaces. This should be done in such a way that the added stresses will assist in improving the conditions of congruence as stated. Obviously, this method of attack, which will correct Lévy's stress regimen, is a marked advance in the design of gravity dams. It is an outstanding contribution to the design of these structures which, perhaps, will yield results of far-reaching consequence.

[53] "Sulla validità del regime Lévy nelle dighe del tipo di gravità," by E. Kalman, *L'Energia Elettrica*, March and April, 1927.

ROBERT E. GLOVER,[54] ESQ. (by letter).[54a]—In Part IV[55] of his paper Mr. Jakobsen states that,

"Instead of the simple Equation (10) for n_y, the condition that Equation (14) must be a minimum is now introduced. This is a problem in variational calculus, but instead of treating it as such, which would be very complicated, a method of approximation may be resorted to, * * *."

The writer has observed that a certain amount of perplexity exists among engineers as to what is to be expected of the variational calculus. This is not surprising since the subject is not included among the courses of study usually prescribed for students in engineering colleges.

It is believed, therefore, that it would be helpful to solve the variational problem presented, in order to have the results available for comparison with those obtained by other methods.

Equation (14) is the expression for the potential energy of deformation. If z = unity, it reduces to,

$$L = \int\int \left[\frac{n_y^2 + n_x^2}{2} - \frac{(n_y + n_x)^2}{2\,(m+1)} + t^2 \right] dy\, dx \qquad (132)$$

It will be convenient to express the stresses in terms of an Airys' function, F, which is connected with the stresses by the relations:

$$n_y = \frac{\partial^2 F}{\partial x^2} - w\,y\ ;\ n_x = \frac{\partial^2 F}{\partial y^2};\ \text{and } t = -\frac{\partial^2 F}{\partial x\,\partial y}$$

This form of expression will be permissible, providing it is consistent with the author's equilibrium Equations (6) and (7). This may be ascertained by direct substitution, as follows:

$$\frac{\partial n_y}{\partial y} = \frac{\partial^3 F}{\partial x^2\,\partial y} - w\ ;\ \frac{\partial n_x}{\partial x} = \frac{\partial^3 F}{\partial x\,\partial y^2};\ \frac{\partial t}{\partial x} = -\frac{\partial^3 F}{\partial x^2\,\partial y};$$

$$\text{and } \frac{\partial t}{\partial y} = -\frac{\partial^3 F}{\partial x\,\partial y^2}.$$

Then,

$$\frac{\partial n_y}{\partial y} + \frac{\partial t}{\partial x} + w = \frac{\partial^3 F}{\partial x^2\,\partial y} - w - \frac{\partial^3 F}{\partial x^2\,\partial y} + w = 0 \qquad (133)$$

$$\frac{\partial n_x}{\partial x} + \frac{\partial t}{\partial y} = \frac{\partial^3 F}{\partial x\,\partial y^2} - \frac{\partial^3 F}{\partial x\,\partial y^2} = 0 \qquad (134)$$

and,

$$L = \int\int \left[\frac{\left(\frac{\partial^2 F}{\partial x^2} - w\,y\right)^2 + \left(\frac{\partial^2 F}{\partial y^2}\right)^2}{2} - \frac{\left(\frac{\partial^2 F}{\partial x^2} - w\,y + \frac{\partial^2 F}{\partial y^2}\right)^2}{2\,(m+1)} + \left(\frac{\partial^2 F}{\partial x\,\partial y}\right)^2 \right] dx\, dy \qquad (135)$$

The writer's computations indicate that if,

$$L = \int\int \phi \left[F, x, y, \frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial^2 F}{\partial x^2}, \frac{\partial^2 F}{\partial y^2}, \frac{\partial^2 F}{\partial x\,\partial y} \right] dx\, dy$$

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[54] Engr., U. S. Bureau of Reclamation, Denver, Colo.

[54a] Received by the Secretary, April 1, 1931.

[55] A statement of the theorem of minimum energy may be found in Paragraph 119 of Love's "Treatise on the Mathematical Theory of Elasticity," Fourth Edition.

the condition for a minimum is,

$$\frac{\partial \phi}{\partial F} - \frac{d}{d x}\left(\frac{\partial \phi}{\partial p}\right) - \frac{d}{d y}\left(\frac{\partial \phi}{\partial q}\right) + \frac{d^2}{d x^2}\left(\frac{\partial \phi}{\partial r}\right) + \frac{d^2}{d y^2}\left(\frac{\partial \phi}{\partial s}\right) + \frac{d^2}{d x\, d y}\left(\frac{\partial \phi}{\partial t}\right) = 0 \quad \text{.......... (136)}$$

in which, $p = \frac{\partial F}{\partial x}$; $q = \frac{\partial F}{\partial y}$; $r = \frac{\partial^2 F}{\partial x^2}$; $s = \frac{\partial^2 F}{\partial y^2}$; $t = \frac{\partial^2 F}{\partial x \partial y}$; and ϕ is a symbol used to denote that the integrand is a function of the variables in the brackets. In the present case, F, x, $\frac{\partial F}{\partial x}$, and $\frac{\partial F}{\partial y}$, are absent, and, therefore,

$$\phi\left[F, x, y, \frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial^2 F}{\partial x^2}, \frac{\partial^2 F}{\partial y^2}, \frac{\partial^2 F}{\partial x \partial y}\right]$$

$$= \frac{\left(\frac{\partial^2 F}{\partial x^2} - w\, y\right)^2 + \left(\frac{\partial^2 F}{\partial y^2}\right)^2}{2} - \frac{\left(\frac{\partial^2 F}{\partial x^2} - w\, y + \frac{\partial^2 F}{\partial y^2}\right)^2}{2\,(m+1)} + \left(\frac{\partial^2 F}{\partial x \partial y}\right)^2$$

Then,

$$\frac{\partial \phi}{\partial r} = \frac{\partial^2 F}{\partial x^2} - w\, y - \frac{\left(\frac{\partial^2 F}{\partial x^2} - w\, y + \frac{\partial^2 F}{\partial y^2}\right)}{(m+1)} \quad \text{........ (137)}$$

$$\frac{\partial \phi}{\partial s} = \frac{\partial^2 F}{\partial y^2} - \frac{\left(\frac{\partial^2 F}{\partial x^2} - w\, y + \frac{\partial^2 F}{\partial y^2}\right)}{(m+1)} \quad \text{.......... (138)}$$

$$\frac{\partial \phi}{\partial t} = 2\, \frac{\partial^2 F}{\partial x \partial y} \quad \text{........................ (139)}$$

and,

$$\frac{d^2}{d x^2}\left(\frac{\partial \phi}{\partial r}\right) = \frac{\partial^4 F}{\partial x^4} - \frac{\left(\frac{\partial^4 F}{\partial x^4} + \frac{\partial^4 F}{\partial x^2 \partial y^2}\right)}{(m+1)} \quad \text{.......... (140)}$$

$$\frac{d^2}{d y^2}\left(\frac{\partial \phi}{\partial s}\right) = \frac{\partial^4 F}{\partial y^4} - \frac{\left(\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4}\right)}{(m+1)} \quad \text{.......... (141)}$$

$$\frac{d^2}{d x\, d y}\left(\frac{\partial \phi}{\partial t}\right) = 2\, \frac{\partial^4 F}{\partial x^2 \partial y^2} \quad \text{.......... (142)}$$

The condition for a minimum states that the sum of Equations (140), (141), and (142), must equal zero; therefore, the condition for a minimum of potential energy of deformation is,

$$\frac{\partial^4 F}{\partial x^4} + 2\, \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0 \quad \text{.............. (143)}$$

Note that the result of the solution of the variational problem is a condition, expressed in terms of a differential equation. This is generally the case.

The problem may be approached from another angle; namely, by basing the argument on the ideas of equilibrium and continuity. As in the previous

case, it will be assumed that Hooke's law applies. Mr. Jakobsen has supplied the equations of equilibrium required, Equations (6) and (7). The equation of continuity, or of compatibility, expresses the condition that if the x, y surface in its unstressed state is marked off into squares by closely spaced lines parallel to the x and y-axes, each of the squares so defined will maintain contact with its neighbors on all four sides after deformation has taken place. The equation of continuity may be arrived at by expressing the strains in terms of the displacements and eliminating the displacements from the equations. To accomplish this let,

u = the displacement of points in the x-direction.
v = the displacement of points in the y-direction.
ϵ_x = the unit strain in the x-direction.
ϵ_y = the unit strain in the y-direction.
γ_{xy} = the unit shearing strain.

Then,

$$\varepsilon_x = \frac{\partial u}{\partial x}\,;\ \varepsilon_y = \frac{\partial v}{\partial y}\,;\ \text{and } \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

Differentiate as follows, and add:

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} = \frac{\partial^3 u}{\partial x\, \partial y^2}$$

$$\frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^3 v}{\partial x^2\, \partial y}$$

$$-\frac{\partial^2 \gamma_{xy}}{\partial x\, \partial y} = -\frac{\partial^3 u}{\partial x\, \partial y^2} - \frac{\partial^3 v}{\partial x^2\, \partial y}$$

then,

$$\frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x\, \partial y} + \frac{\partial^2 \varepsilon_x}{\partial y^2} = 0 \ldots\ldots\ldots\ldots (144)$$

Equation (144) is the required equation.[56]

Introduce Hooke's law in terms of the ordinates to Airy's surface, as follows:

$$\varepsilon_x = \frac{1}{E}\,(n_x - \mu\, n_y) = \frac{1}{E}\left(\frac{\partial^2 F}{\partial y^2} - \mu\,\frac{\partial^2 F}{\partial x^2} + \mu\, w\, y\right) \ldots (145)$$

$$\varepsilon_y = \frac{1}{E}\,(n_y - \mu\, n_x) = \frac{1}{E}\left(\frac{\partial^2 F}{\partial x^2} - w\, y - \mu\,\frac{\partial^2 F}{\partial y^2}\right) \ldots\ldots (146)$$

$$\gamma_{xy} = \frac{2\,(1+\mu)\, t}{E} = \frac{2\,(1+\mu)}{E}\left(-\frac{\partial^2 F}{\partial x\, \partial y}\right) \ldots\ldots\ldots\ldots (147)$$

Note that, in Equations (145), (146), and (147), $\mu = \frac{1}{m}$. If these expressions are substituted into the equation of continuity (Equation (144)):

$$\frac{\partial^4 F}{\partial x^4} + 2\,\frac{\partial^4 F}{\partial x^2\, \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0 \ldots\ldots\ldots\ldots\ldots (148)$$

As a result of this work it may be seen that, not only does Equation (148) imply that the equations of compatibility, as stated by the author in Appendix

[56] Compare Love's "Mathematical Theory of Elasticity," Paragraph 17, Fourth Edition.

II, are satisfied, but it also implies the satisfaction of the equations of equilibrium. Furthermore, the variational calculus indicates that if this equation is satisfied the potential energy of deformation will be less than that of any other distribution whatever, which will hold the forces in equilibrium.

A specification may be written, therefore, for a solution of the problem of finding the stress distribution in a two-dimensional elastic structure. The specification is, solve Equation (148) subject to the boundary conditions. This test may be applied to the solutions in the author's paper. Equations (13) satisfy Equation (148). That Equations (27) do not, is not conclusive in this case, since the author does not pretend that these equations represent an exact solution. Neither Equations (13) nor (27), however, recognize explicitly any elastic action of the foundation and, therefore, it must be concluded that neither is entitled to acceptance as a true solution of the problem. The objection may be stated specifically in the case of Equations (27). The limits given for the integrals in Equations (19) and (20) show that the integration was carried over the section of the dam only. This being the case, one may conclude that Mr. Jakobsen has succeeded in solving, approximately, the problem:

To find, among all possible stress distributions which satisfy the boundary conditions at the up-stream and down-stream faces of a triangular profile, that distribution which shall make the potential energy of deformation within the section of the dam a minimum.

The problem to be solved in case the law of least work is to be complied with, is:

To find among all possible stress distributions which satisfy the boundary conditions at the surface of the dam and foundation, that distribution which shall make the total potential energy of deformation in the dam and foundation, a minimum.

P. WILHELM WERNER,[57] ASSOC. M. AM. SOC. C. E. (by letter).[57a]—The author's contribution to the development of the theory of gravity dams is interesting. Especially in view of the tendency toward ever-increasing height of dams, it is important that the methods on which the design is based should be reviewed. It is well known that the theory now generally used is not strictly correct from the standpoint of the general theory of elasticity.

In discussing the paper, the writer wishes to call attention to an aspect of the problem that has special relation to one of the boundary conditions at the base of the dam.

Under the usual assumption that,

$$n_z = t_{zy} = t_{zx} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (149)$$

and that the material in the dam is homogeneous and isotropic and follows Hooke's law, the exact solution of the problem involves finding a stress function, F, which satisfies the homogeneous partial differential equation:

$$\frac{\partial^4 F}{\partial x^4} + 2 \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0 \ldots\ldots\ldots\ldots (150)$$

and which, furthermore, satisfies all the boundary conditions, when the co-

[57] Aktiebalaget Vattenbyggnadsbyran, Stockholm, Sweden.

[57a] Received by the Secretary, February 14, 1931.

ordinates of the edges of the cross-section are introduced into the following general expressions for the stresses, n, and the displacements, u:

$$n_x = \frac{\partial^2 F}{\partial y^2} \qquad (151)$$

$$n_y = \frac{\partial^2 F}{\partial x^2} \qquad (152)$$

$$t = -\frac{\partial^2 F}{\partial x \, \partial y} - w\,x \qquad (153)$$

$$\frac{\partial u_x}{\partial x} = \frac{1}{E}\left(n_x - \frac{1}{m}\,n_y\right) \qquad (154)$$

$$\frac{\partial u_y}{\partial y} = \frac{1}{E}\left(n_y - \frac{1}{m}\,n_x\right) \qquad (155)$$

As pointed out by the author, the solution given in Equations (27) does not satisfy the general differential equation, Equation (150). As regards the boundary conditions, it is evident that neither Equations (27) nor Equations (13) fulfill the conditions for the displacements at the base of the dam. The author discusses the condition for u_y, that is, the displacement in the direction of the y-axis, but he does not discuss the condition for u_x, that is the displacement in the direction of the x-axis. In the writer's opinion, however, the boundary condition for u_x at the base is also of rather great importance.

The first conception one has regarding the boundary condition for u_x at the base is that,

$$\left.\frac{\partial u_x}{\partial x}\right]_{y = H} = 0 \qquad (156)$$

that is, that all the points of the base, under loaded conditions, retain their positions in relation to each other in the direction of the x-axis. The extent to which this is actually true, is, of course, impossible to determine. A certain foundation deformation may exist even in the direction of the x-axis. Such deformation (if it does exist), may be influenced by irregularities in the foundation, etc. However, the strain condition expressed by Equation (156) should be found more nearly to correspond to the actual conditions at the base of the dam than any arbitrarily chosen distribution of the shearing stress, and, in any case, Equation (156) must be considered as a possible limiting case.

It is easy to verify the fact that neither Equations (13) nor Equations (27) satisfy the boundary condition expressed by Equation (156). In spite of this, both theories give (in accordance with St. Venant's principle) very nearly correct values for the stresses in points at some distance from the base of the dam. Near the base, however, there might be considerable discrepancy between the actual stress conditions and the stresses computed according to either theory, as long as the boundary conditions along the base are not fulfilled. Unfortunately, the inaccuracy of the methods applies to the most vulnerable part of the dam, that is, the base.

It is interesting to compare the values of the strain, $\varepsilon = \frac{\partial u_x}{\partial x}$, at the base, computed on the basis of Equations (13) and Equations (27). In a no-uplift section and under the assumption that $m = \infty$, the following values are determined by substitution in Equation (154):

(1) By the straight-line theory:

$$\varepsilon_1 = \frac{1}{E} g\,H = -0.4340 \frac{H}{E} \qquad (157)$$

(2) By the author's least work theory:

$$\varepsilon_2 = -0.4340 \frac{H}{E} + k^4 z^4 (1.584 - 1.579\,k\,z) \frac{H}{E} \qquad (158)$$

in which, $z = \frac{x}{b}$.

Evidently, for $k < \frac{1.584}{1.579}$ — that is, if k is less than about unity — the numerical value of ϵ_2 is always less than that of ϵ_1, except at the up-stream face ($z = x = 0$), where the two values are exactly equal, or,

$$\varepsilon_1 = \varepsilon_2 = -0.4340 \frac{H}{E} \qquad (159)$$

At the down-stream face of the dam ($x = b$, $z = 1$), the difference between ϵ_2 and ϵ_1 is greatest, amounting to about 22% for $k = 0.645$.

In the writer's opinion a revision of the theory for gravity dams, and for buttresses of hollow dams as well, must aim at the fulfillment of the boundary condition for u_x at the base. The author's trial function, as expressed by Equations (27), leads to considerably more dangerous stresses than the trial function according to the old theory, as expressed by Equations (13). However, the writer has shown in the foregoing remarks that the author's least work theory affects the boundary condition for u_x at the base only to a very moderate extent. If the same proportion for a complete fulfillment of the boundary condition for u_x at the base exists, it would seem to indicate disastrous stresses in the dam. The writer is fully aware that this same proportion might not exist, but the question certainly seems worth while investigating.

In general, it is very difficult, if not impossible, to find, in a given case, the solution of the general differential Equation (149) which at the same time satisfies all the boundary conditions. As a rule the engineer must be content with an approximate solution, such as that given by the Equations (27). Such approximate solutions must be carefully viewed and discussed, especially in regard to the boundary conditions.

In this special case the writer would suggest that the method outlined in the paper be verified in such a way that it can be applied to another problem of similar nature the exact solution of which, from the standpoint of the general theory of elasticity, is known beforehand.

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DISCUSSIONS

FORMULAS FOR RAINFALL INTENSITIES OF LONG DURATION

Discussion

BY MESSRS. R. W. POWELL, C. E. GRUNSKY, AND CHARLES W. SHERMAN

R. W. POWELL,[7] ASSOC. M. AM. SOC. C. E. (by letter).[7a]—The author's charts give in condensed form a mass of valuable information regarding the probable intensity of rainfall for periods of from 2 hours to 3 days for an important section of the United States. It is to be hoped that some one will develop similar information for the remainder of the country. It is questionable, however, whether it is worth while to distinguish different values of n for different frequencies because, for most places, the values in Fig. 11 (for a frequency of once in twenty-five years) could be used for all frequencies without introducing errors larger than those which are probably already present in the data. For example, the values of n shown on the charts for Detroit, Mich., vary from 0.780 to 0.805, while Milton F. Wagnitz, M. Am. Soc. C. E., and Lewis C. Wilcoxen, Assoc. M. Am. Soc. C. E., from a study of Detroit data alone found a value of 0.855.[8] On the other hand, C is clearly a function of the frequency, but it seems probable that this relationship could be represented by an equation which would require the use of only one map. Charles W. Sherman,[9] M. Am. Soc. C. E., has suggested a formula for Boston, Mass., which can be expressed as:

$$C = k\,F^{0.27} \quad \text{.......................... (4)}$$

Equation (10) for Detroit[8] is practically the same formula. The writer has read values for Columbus, Ohio, from the author's charts and finds a relationship that can be expressed either by,

$$C = 24\ F^{0.23} \quad \text{.......................... (5)}$$

or

$$C = 13 + 27 \log_{10} F \quad \text{.......................... (6)}$$

Equation (6) gives the best fit, but Equation (5) is the more convenient for combining with other equations. Putting it in the general form,

$$C = k\ F^{x} \quad \text{.......................... (7)}$$

NOTE.—The paper by Merrill M. Bernard, M. Am. Soc. C. E., was published in October, 1930, *Proceedings*. Discussion on the paper has appeared in *Proceedings* as follows: February, 1931, by Messrs. W. B. Gregory, W. W. Horner, and Adolph F. Meyer.

[7] Asst. Prof. of Mechanics, Ohio State Univ., Columbus, Ohio.

[7a] Received by the Secretary, March 2, 1931.

[8] *Proceedings*, Am. Soc. C. E., December, 1930, Papers and Discussions, p. 2180.

[9] *Loc. cit.*, April, 1930, Papers and Discussions, p. 721.

and substituting in Equation (3):

$$i = \frac{k F^x}{t^n} \quad \text{..........................} (8)$$

For long periods the total precipitation is more useful than the rate of precipitation. If P = total inches of precipitation in t minutes,

$$P = \frac{t\ i}{60} = \frac{k\ t^{(1-n)}\ F^x}{60} = K\ t^{(1-n)}\ F^x \quad \text{..........} (9)$$

in which, $K = \frac{k}{60}$. For most points considered n seems to be approximately 0.75 and x, approximately 0.25, which leads to the interesting approximate formula,

$$P = K\ (t\ F)^{0.25} \quad \text{.......................} (10)$$

This would require only one map giving values of K. For more accurate values Equation (9) would be used. This would require another map giving values of $(1 - n)$. The coefficient, x, would probably vary so little with the location that an average value could be used.

The question presents itself as to whether these formulas could be used for periods longer than three days. This was tested for Columbus, Ohio, by computing the values given for a period of one month and comparing them with observed values. Using $n = 0.77$, $k = 24$, and $x = 0.23$, Equation (9) becomes,

$$P = 0.40\ t^{0.23}\ F^{0.23} \quad \text{...................} (11)$$

TABLE 4.—PROBABLE MAXIMUM MONTHLY PRECIPITATION AT COLUMBUS, OHIO

Frequency of occurrence once in F years (1)	Total precipitation, P, in inches, by Equation (11) (2)	From the curve of observed values, Fig. 16 (3)
5	6.77	7.7
10	7.94	8.4
15	8.71	9.0
25	9.80	9.8
50	11.49	10.9
100	13.48	12.1

The equality of the exponents in Equation (11) is probably only a coincidence. Taking the average month as 43 830 min., the probable maximum monthly precipitation occurring once in F years is computed in Table 4. Values in Column (3), Table 4, were taken from Fig. 16. These curves were plotted from records of the maximum recorded monthly rainfall at the Columbus Weather Bureau from 1878 to 1921, at Ohio State University from 1882 to 1921, and at Westerville (13 miles north of Columbus), from 1854 to 1902.[10] The data were combined to make one composite record of a total of 1 555 months which was considered as typical of the vicinity.

[10] Data from "A Climatological History of Ohio," by W. H. Alexander, pub. by the Eng. Experiment Station, Ohio State Univ., 1923.

The fact that these formulas give such reasonable values when extrapolated so far beyond the range for which they were derived, speaks well for the soundness of the author's method and indicates that his formulas could be used safely for periods of more than three days. It is interesting to note,

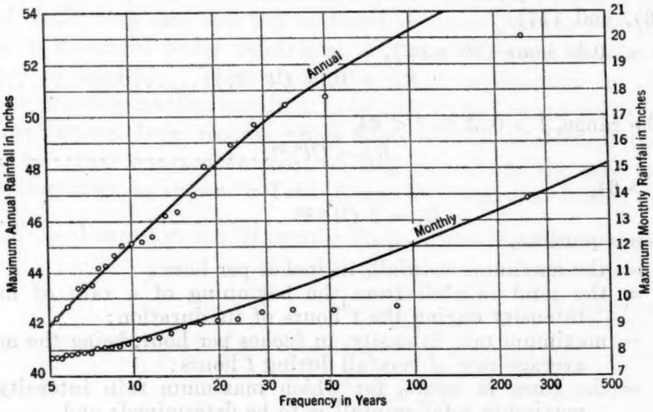

FIG. 16.—MAXIMUM RAINFALL AT COLUMBUS, OHIO.

however, that the formulas do not hold for periods as long as one year. Equation (11) gives a 1-year maximum of 12.0 in. once in 5 years, and 23.9 in. once in 100 years. The actual values from Fig. 16 are 42.0 and 53.3 in., respectively. These conclusions are based on Columbus, Ohio, only and need confirmation from other points.

C. E. GRUNSKY,[11] PAST-PRESIDENT, AM. SOC. C. E. (by letter).[11a]—As the outcome of long experience in the use of formulas for the approximation of probable maximum rain intensity during time periods not sufficiently covered by actual observation, the writer suggests the following formulas which differ from those heretofore in common use mainly in that the factor, time, is expressed in hours instead of in minutes.

This departure appears desirable because the rain intensity is commonly expressed in inches per hour and not in inches per minute. Furthermore, the coefficient which appears in the formula most frequently used, thus becomes equal to the maximum number of inches of rain in 1 hour, a fact which facilitates a first approximation thereof. The formulas herein suggested should commend themselves by reason of their simplicity.

The maximum rain intensity during t hours, can be expressed by Equations (12), (13), and (14):

For $t < 0.33$ hour (20 min.),

$$i = \frac{0.75\, C}{t^{0.75}} \quad \text{(12)}$$

For the range, $t > 0.33$ to $t < 64$,

$$i = \frac{C}{t^{0.50}} \quad \text{(13)}$$

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[11] Cons. Engr. (C. E. Grunsky Co.), San Francisco, Calif.

[11a] Received by the Secretary, March 3, 1931.

For $t > 64$,

$$i = \frac{2\ C}{t^{0.67}} \qquad (14)$$

The maximum rain intensity during t hours, can be expressed by Equations (15), (16), and (17):

For $t < 0.33$ hour (20 min.),

$$R_t = 0.75\ C t^{0.25} \qquad (15)$$

For the range, $t > 0.33$ to $t < 64$,

$$R_t = C t^{0.50} \qquad (16)$$

For $t > 64$,

$$R_t = 2\ C t^{0.33} \qquad (17)$$

In these equations,

R = the maximum rainfall, in inches per hour;

R_t = the total rainfall from the beginning of a rain of maximum intensity during the t hours of its duration;

i_t = maximum rain intensity, in inches per hour, being the maximum average rate of rainfall during t hours;

t = the time, in hours, for which maximum rain intensity or the maximum total rainfall is to be determined; and

C = a coefficient which is to be ascertained for any locality from records of measured maximum rainfall.

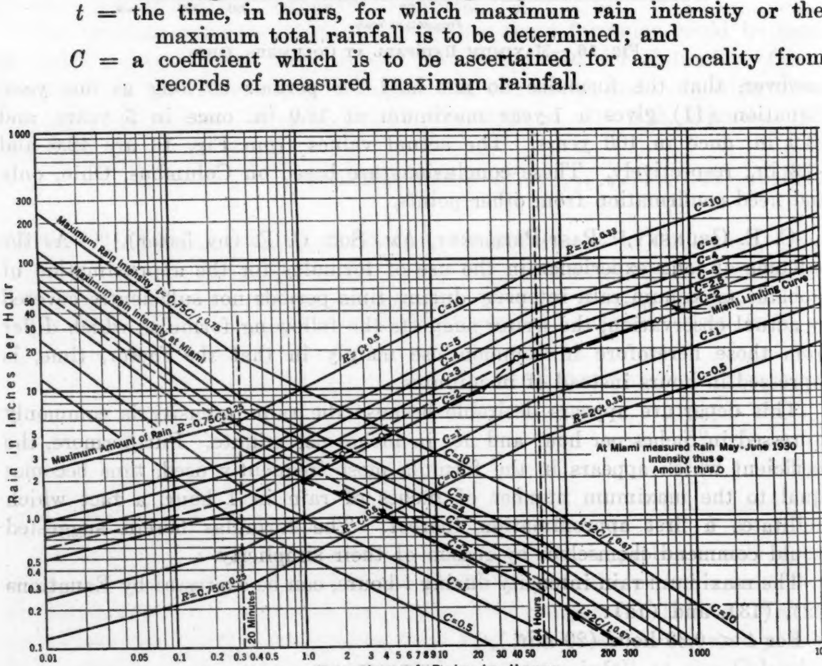

FIG. 17.—PROBABLE RAINFALL MAXIMA IN VARIOUS TIME PERIODS AND CURVES OF MAXIMUM RAIN INTENSITY.

These formulas for maximum rain intensity as well as for the maximum quantity of rain, if plotted, will be found to represent continuous lines with slight changes in direction at the 0.33-hour and 64-hour points. Fig. 17 shows

these lines on logarithmic paper for values of t, ranging from 0.01 hour (0.60 min.) to periods far beyond any that ordinarily come under consideration.

The formulas are to be adapted to any place by the selection of a proper value of C. In each case this will be based, of course, on the best information relating to measured heavy rainfall at, or near, or in the general region of, the place in question. They should be found satisfactory in all cases in which the probable maximum rain is to be determined at any place for short or long time periods, from records which have already established the probable maxima for other comparable short or long periods, as the case may be. A computation sheet, as shown in Table 5, will be found convenient.

TABLE 5.—FORMULAS FOR MAXIMUM RAIN INTENSITY, COMPUTATION SHEET

Time, in hours, t	$t^{0.25}$	$t^{0.50}$	$t^{0.33}$	$\frac{1}{t^{0.75}}$	$\frac{1}{t^{0.50}}$	$\frac{1}{t^{0.67}}$	Maximum Rain Intensity, in Inches per Hour			Maximum Quantity of Rain, in Inches		
							$\frac{0.75\ C}{t^{0.75}}$	$\frac{C}{t^{0.50}}$	$\frac{2\ C}{t^{0.67}}$	$0.75\ C\ t^{0.25}$	$C\ t^{0.50}$	$2\ C\ t^{0.33}$
0.0167	0.36			21.5			16.1 C			0.27 C		
0.083	0.54			6.5			4.7 C			0.40 C		
0.167	0.64			3.8			2.9 C			0.48 C		
0.333	0.76			2.28			1.71 C			0.57 C		
0.50		0.71			1.41			1.41 C			0.71 C	
1		1.00			1.00			1.00 C			1.00 C	
2		1.41			0.71			0.71 C			1.41 C	
3		1.73			0.58			0.58 C			1.73 C	
4		2.00			0.50			0.50 C			2.00 C	
5		2.24			0.447			0.45 C			2.24 C	
6		2.45			0.408			0.41 C			2.45 C	
12		3.46			0.289			0.29 C			3.46 C	
24		4.90			0.204			0.20 C			4.90 C	
48		6.94			0.144			0.14 C			6.94 C	
64		8.00			0.125			0.125 C			8.00 C	
72			4.13			0.0585			0.117 C			8.26 C
96			4.58			0.0475			0.095 C			9.16 C
120			4.94			0.0410			0.082 C			9.88 C
240			6.21			0.0259			0.052 C			12.42 C
720			8.97			0.0124			0.025 C			17.94 C

These formulas are intended to replace formulas of the types (Equations (2) and (3)) referred to by the author. The coefficients a, b, and C, and the exponent, n, in Mr. Bernard's formulas, all depend for value on locality. The time, t, in these formulas is expressed in minutes while i is expressed in inches per hour.

The simplified Equations (12) to (17) are intended to obviate the inconvenience and limited application of the older types.

The following examples of exceptionally heavy rainfall are illustrative:

Example 1.—At Campo, Calif., on August 12, 1891, 11.5 in. of rain fell in 1.33 hours (80 min.). What was the probable maximum rainfall in 1 min. during this storm?

In this case, because $t > 0.33$ and < 64, Equation (13) applies: $i_t = \frac{11.5}{1.33} = 8.65 = \frac{C}{1.33}$; $C = 8.65 \times 1.33 = 10.0$ (in round numbers); and for $t = 1$ min.

$= 0.0167$ hour (by Equation (16)), $R_{0.0167} = 0.75 \times 10 \times 0.0167^{0.25} = 7.5 \times 0.36 = 2.7$ in., the probable maximum in 1 min.

Example 2.—At Orpid's Camp, about 20 miles northeasterly from Los Angeles, Calif., observations with two recording rain gauges, about 4 ft. apart, showed rainfall in 1 min. (from 4:43 to 4:44 A. M.) of 1.03 in. in one gauge and 0.92 in. in the other. What coefficient does this indicate for the formulas of maximum rainfall?

Taking the maximum rainfall of this storm at 1.00 in. per min. (0.0167 hour), $i_{0.0167} = 1.00$ in. per min. $= 60$ in. per hour. By Equation (15), $R_{0.0167} = 1 = 0.75\, C\, (0.0167^{0.25})$; from which, $C = 4.2$ and $R = 4.2$ in., the probable maximum for any storm at this place during 1 hour.

The storm produced 1.17 in. of rain in 10 min. and 2.20 in. in 1 hour. For the 10-min. period (0.167 hour) Equation (15) would give, $R_{0.167} = 4.2 \times 0.167^{25} = 2.7$ in., the probable maximum, to be compared with the 1.17 in. as measured.

TABLE 6.—RAIN AT MIAMI, FLORIDA, MAY 27 TO JUNE 19, 1930, COMPARED WITH COMPUTED MAXIMA

Period, hours	Measured Rainfall		Computed by Formulas		Remarks, $C = 2.5$
	Intensity, in inches per hour	Quantity, in inches	Maximum intensity, in inches per hour	Maximum quantity, in inches	
0.0167	.....	......	40.	0.67	1 min.
0.167	.....	......	7.2	1.20	10 min.
0.50	.....	......	3.5	1.77	30 min.
1.	1.87	1.87	2.50	2.50	....
2	1.54	3.07	1.77	3.54	....
3	.....	......	1.44	4.33	....
4	1.10	4.39	1.25	5.00	....
5	0.91	4.53	1.12	5.59	....
6	.....	......	1.02	6.13	....
12	.....	......	0.94	8.65	....
24	0.38	9.36	0.51	12.25	....
44	0.375	16.49	0.378	16.60	43.93 hours.
48	.....	......	0.360	17.3	....
72	.....	......	0.286	20.6	....
96	.....	......	0.242	23.2	....
120	0.161	19.28	0.205	24.6	....
240	.....	.....	0.108	26.	....
545*	0.061	33.16*	0.075	41.	....
720	.....	......	0.062	45.	30 days.
1 440	.....	......	0.0395	57.	....
8 640	.....	......	0.0119	103.	1 year.

*This period from May 27 to June 19, includes 49 consecutive hours on May 27, 28, and 29, after the beginning of the storm in which no rain fell.

Example 3.—In a letter to the writer, Mr. J. F. Voorhees, Associate Meteorologist of the U. S. Weather Bureau, has stated that:

"On November 18, 1930, an automatic recorder belonging to the U. S. Geological Survey and situated in Moanalua Valley, Honolulu, measured 15.2 in. in 3 hours and 5.6 in. in 1 hour."

In this case, $C = 5.6$, and the probable maximum fall of rain during any single minute was $5.6 \sqrt[3]{0.0167} = 1.4$ in.

Example 4.—At Miami, Fla., there was an unusually heavy rain from May 29 to June 2, 1930.[12] This storm established a near record for the periods of 1 hour, 2 hours, 4 hours, and a full day, as well as for the longer period of 44 hours. A comparison of the measured rain, during this storm, with the limiting curve deduced therefrom, is presented in Table 6. In this table, R_{44} (43.93 hours), equals 16.49 in. Then, by Equation (16), $R_{44} = C\sqrt{43.93} = 16.49$; from which, $C = 2.48$. Similarly, $R_4 = C\sqrt{4} = 4.39$; from which, $C = 2.20$.

These results indicate that at Miami the value of C should be taken at 2.50. It is with this value that the comparisons shown in Table 6 have been made.

It appears from the foregoing that while the greatest measured rainfall in 1 min. may well be credited to Orpid's Camp, there have, no doubt, been many other rainstorms with short-time rainfall of still greater intensity, as indicated by Examples 1 and 3.

More is known of the maximum rate of precipitation to be expected at single stations than of the maximum average rate in any selected period of time over extensive water-sheds. Thus, for example, during the entire storm at Campo in which 11.5 in. of rain fell in 80 min., the rainfall at another point only 1.25 miles distant, was only 3 in. What correction factors should be used for area and for location and orographic features? There is still ample field for study and such contributions as that of Mr. Bernard are valuable additions to the literature on this subject.

CHARLES W. SHERMAN,[13] M. AM. SOC. C. E. (by letter).[13a]—In utilizing the Miami Conservancy District records of 1-day, 2-day, 3-day storms, etc., allowance ought to be made for the fact that few if any of these storms lasted exactly 24, 48, or 72 hours. Nearly all the figures are based on records of ordinary (not recording) rain-gauges, observed once daily. All the rain accumulated between two such readings is assumed to have fallen in 24 hours. There is nothing in the records themselves to show that a 1-day precipitation may not have fallen in a single hour or even less, a 2-day collection in, say, 2 hours, one preceding and one following the time of observation, and a 3-day collection within about 25 or 26 hours. It is unlikely that maximum rainfalls can occur in such short periods, but it is not improbable that some of the high 1-day rainfalls actually occurred in periods not exceeding 20 hours; the 2-day rainfalls in 40 to 42 hours, etc.

The writer believes, therefore, that Mr. Bernard was not justified in using 1 440 min., 2 880 min., etc., as representing the durations of such storms. In no case could the durations exceed those assumed, and in an appreciable number of cases they must have been considerably shorter. Would it not have been more correct to assume that the actual durations were at least 10% shorter than the number of minutes corresponding to the length of the storm in days? Actually, the storms must have varied in duration, so that

[12] See *Monthly Weather* Review, June, 1930.

[13] Cons. Engr. (Metcalf & Eddy), Boston, Mass.

[13a] Received by the Secretary, March 23, 1931.

curves based upon the assumption that all 1-day storms lasted exactly 1440 min., or any other definite number, can be at best only approximate.

Another inaccuracy is in the treatment of all rainfall records within an arbitrary quadrangle, as though they represented observations at a single station, with a length of record equal to the sum of the periods of observation at the several stations. In the first place, the quadrangles are so large that wide variations in storm-producing conditions are likely to exist at various points in many of them; and, second, the combination of a number of short records obtained during a period when either an excess or a deficiency in rainfall is generally prevalent, cannot possibly be equivalent even to one long-term record covering a wide range of climatological conditions. In the major part of the United States, such a wide range is experienced when the term of years is great enough. The writer does not mean to imply that no advantage results from the combination of records, but rather to call attention to the fact that implicit confidence should not be placed in the formulas derived.

Examination of the charts, for values of the exponent, n (Figs. 5, 7, 9, 11, 13, and 15) shows a comparatively small range in these values—roughly from 0.72 to 0.83. In view of the inaccuracies inherent in the records, and the method of combining them, it appears to the writer that a single value, and that a convenient fraction, such as $\frac{3}{4}$ or $\frac{4}{5}$, might have been assumed for n, which would leave only one factor varying with location, namely, the coefficient, C.

The writer has shown elsewhere[14] that at least for some localities an intensity-duration equation of the form,

$$i = \frac{a}{(t+b)^n} \qquad (18)$$

is much better for storms of short duration when the exponent approximates 0.7 than the Meyer form in which the exponent is 1.0. If b is less than 10, as seems probable from such data as have been studied, this does not differ materially in the case of long storms from,

$$i = \frac{a}{t^n} \qquad (19)$$

As a matter of fact, the writer found a single formula applicable to Boston storms of both long and short durations, at least up to about 30 hours. With longer storms, especially those of rare occurrence, the formula was not as good a representation of the observed intensities as might be desired.

These criticisms are intended primarily as warnings against putting too great faith in the formulas derived by Mr. Bernard. They are not intended to throw discredit upon the results for practical use in drainage problems. The immense amount of detailed work which the author has devoted to the study is obvious, and the writer believes that, for the section of the country to which the figures are applicable, the available data have been arranged in such shape that the designer of drainage works can make use of them with much greater confidence than has been possible heretofore.

[14] *Proceedings*, Am. Soc. C. E., April, 1930, Papers and Discussions, p. 717.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852

DISCUSSIONS

THE BUTTRESSED DAM OF UNIFORM STRENGTH

Discussion

By Messrs. Eugene Kalman, A. Floris, Fredrik Vogt, and A. C. Schwager

Eugene Kalman,[16] M. Am. Soc. C. E. (by letter).[16a]—Although the writer cannot agree with all the conclusions drawn by Mr. Schorer, he considers this paper a valuable contribution to the study of dam design. The ordinary buttress shape defined by two more or less parallel planes has been discarded and replaced by a more complicated form. The choice of that form constitutes a remarkably ingenious effort, and it proves that sometimes by making structures more complicated it is possible to simplify their analysis due to the special shape.

Arch Column Unit.—The author's analysis is rather difficult to understand, and it will be useful, therefore, to review some of the underlying principles briefly. In Fig. 1 the two loads, P_0 and P_1, acting on the upper and lower section of the arch column, are necessarily the closing sides of the string polygon formed by the axis of the column, the intermediate forces being represented by the weights of the single elements in the column. Since all those weights are acting vertically downward, the force polygon will have the shape illustrated in Fig. 26(*b*). The vectors, $O\,B$ and $O\,D$, represent the magnitude and direction of the end loads, P_0 and P_1. The vector, $O\,C$, represents the resultant acting at a point along the axis of the column at which the tangent to the axis is parallel to $O\,C$, as shown in Fig. 26(*a*).

First, the shape of the column axis must be known and, therefore, its equation must be determined. This may be accomplished by equating the inclinations of an arbitrary intermediate force in the funicular polygon and in the force polygon. Let ϵ and η denote the co-ordinates of the column axis. Then, the slope of the force, P, in Fig. 26(*a*), may be expressed by,

$$\tan \phi = -\frac{d\eta}{d\epsilon} \qquad (38)$$

Note.—The paper by Herman Schorer, Assoc. M. Am. Soc. C. E., was published in November, 1930, *Proceedings*. Discussion of the paper has appeared in *Proceedings* as follows: January, 1931, by Messrs. James Girand and Fred A. Noetzli; February, 1931, by Messrs. B. F. Jakobsen and Calvin V. Davis; and March, 1931, by A. A. Eremin, Assoc. M. Am. Soc. C. E.

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[16a] Received by the Secretary, February 24, 1931.

On the other hand, the inclination of $O\,C$ is expressed as follows:

$$\tan \phi = \frac{A\,C}{O\,A} = \frac{A\,B + B\,C}{O\,A} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (39)$$

Since $B\,C$ is the total weight of the column between the points of application of P_0 and P, its value will be given by the sum,

$$w = \gamma_c \int_{\varepsilon_0}^{\varepsilon} w\,t\,ds \ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

in which, w, t, and ds are the width, the thickness, and the elementary arch length of the column, respectively, at an arbitrary point on the axis.

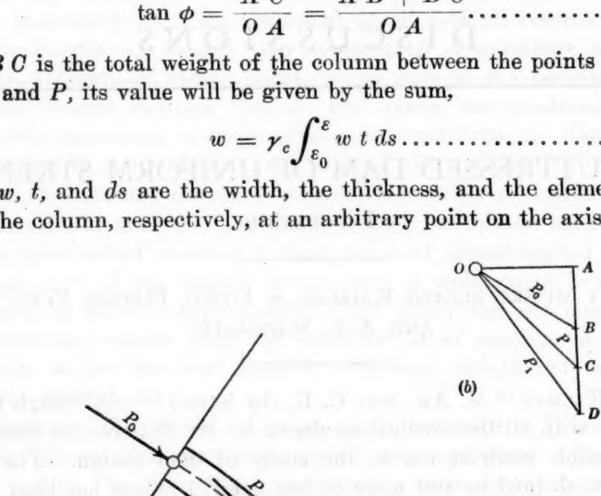

FIG. 26

The values of $O\,A$ and $A\,B$ are determined by the magnitude and direction of the upper end load, P_0, and hence from Equations (38) and (40),

$$\frac{P_0 \sin \phi_0 + \gamma_c \int_{\varepsilon_0}^{\varepsilon} w\,t\,ds}{P_0 \cos \phi_0} = \frac{d\eta}{d\varepsilon} \ldots\ldots\ldots\ldots (41)$$

Equation (41) expresses one of the two fundamental relations of the problem. The other follows from the specification of a constant unit compressive stress, f_c, at all sections of the column. That stress may be obtained by dividing the force, P, by the area, $w\,t$, on which it is distributed. The magnitude of P can be evaluated from Fig. 26(b) as follows:

$$P = O\,A \sec \phi = P_0 \cos \phi_0 \sec \phi \ldots\ldots\ldots\ldots (42)$$

from which the second fundamental relation is,

$$\frac{P_0 \cos \phi_0 \sec \phi}{w\,t} = f_c \ldots\ldots\ldots\ldots\ldots\ldots\ldots (43)$$

In order to obtain the equation of the axis, both sides of Equation (41) must be differentiated. This operation may be performed after the substitution of the arch element, ds, by $\sec \phi \, d\varepsilon$, thus deriving the equation,

$$-\frac{d^2 \eta}{d \varepsilon^2} \frac{P_0 \cos \phi_0}{\gamma_c} = w \, t \sec \phi \ldots\ldots\ldots\ldots\ldots (44)$$

The product, $w \, t$, may be evaluated from Equation (43) for use in Equation (44), with the following result:

$$-\frac{d^2 \eta}{d \varepsilon^2} \frac{P_0 \cos \phi_0}{\gamma_c} = \frac{P_0 \cos \phi_0 \sec^2 \phi}{f_c} \ldots\ldots\ldots\ldots\ldots (45)$$

and applying the well-known formula,

$$\sec^2 \phi = 1 + \tan^2 \phi = 1 + \left(\frac{d\eta}{d\varepsilon}\right)^2 \ldots\ldots\ldots\ldots\ldots (46)$$

the ultimate form of Equation (45) will be:

$$c \frac{d^2 \eta}{d\varepsilon^2} + \left(\frac{d\eta}{d\varepsilon}\right)^2 + 1 = 0 \ldots\ldots\ldots\ldots\ldots (47)$$

This is a differential equation of the second order, for which the following general solution applies:

$$\eta = c \log \cos \frac{\varepsilon + C_1}{c} + C_2 \ldots\ldots\ldots\ldots\ldots (48)$$

A simple substitution will show that Equation (48) is really an integral of Equation (47), in which, C_1 and C_2 are two arbitrary constants. Equation (48), expressing the curve of the column axis, indicates that two geometrical conditions may be imposed upon it, and, since the position of the upper end is always given, as well as the slope of the up-stream face which yields the first derivative of the catenary, the curves of Equation (48) are entirely determined. With the form of the up-stream face given, there is one, and only one, catenary that passes through each point of that curve. The buttress is, then, subdivided into a certain number of independent catenaries, for which the width, w, is known at every point. Consequently, Equation (43) enables the designer to select a thickness at every section of each independent column unit, such that the unit stress at any section will remain constant.

Multiple Column Buttress of Uniform Strength.—A structure is obtained by the foregoing method that represents an ideal buttress. At every section the stress is constant, provided the columns are not in contact with each other and the proper loads, P_0 and P_1, are applied at their upper and lower ends.

Obviously, this ideal structure cannot be duplicated in actual practice. In the first place, the independent column units cannot be built; and, in the second place, the practical buttress will have to rest on the foundation and not on a system of needles, such as P_1.

In constructing the buttress one column unit must be placed upon the other; and after the construction, when the reservoir is filled, the column units will undergo certain deformations. Will they act independently of each other? In a buttress built as a monolithic structure, will the trajectories follow the ideal catenary curves? The influence of the minor principle stress has a

tendency to disturb the equilibrium of the individual column units. Will the effect of this tendency be negligible?

These are the questions that are raised by a study of the author's method and there is no satisfactory answer to them in his paper. Unfortunately, Mr. Schorer makes extensive use of a refined mathematical tool to analyze the case of a buttress subdivided into column units and to explain their statical behavior (which does not interest the designer very much). On the other hand he drops the theoretical considerations entirely in treating the reconstruction of the buttress from the column units and in explaining the statical behavior of the buttress (which is the only thing that really does interest the designer).

The author merely limits himself to a few generic remarks which are absolutely not convincing. He writes, for instance:

"Since the buttress is subject mainly to direct stresses in one direction, the internal work must approach a possible minimum. From Castigliano's principle of least work it can then be concluded that the stress distribution in the monolithic buttress is very nearly the same as that in the multiple column buttress composed of independent arched column units."

The foregoing statement is nebulous and the writer has been entirely unable to interpret it. The paper does not state whether the foundation will re-act exactly or approximately in conformity with the load system, P_1, which was selected, without any consideration of the foundation. It is assumed that the catenaries are trajectories and that the minor principle stresses are negligible everywhere; but these assumption have not been proved by the author.

Comparison with the Principle of Linear Stress Distribution.—Since the paper does not include a quantitative analysis of the stresses in the proposed buttress, the most simple check at hand might be attempted, and that is to compare it with the principle of linear stress distribution at a horizontal section of the buttress.

In the case of a plane up-stream face, all catenaries are identical curves, since $\phi_0 = 0$. The lower catenaries are obtained from the uppermost one by a simple translation. This fact follows since that portion of the upper catenary unit lying to the right of any arbitrary section may be removed and the corresponding thrust, P, at that section may be substituted for it.

The only difference between the remaining portions of the highest catenary unit and the lower column units is that the upper end-points of the first are acted upon by the constant, P_0, while the load, P_0, acting on the upper ends of the lower column units will depend on the depth. This will involve a corresponding increase in the thickness in the lower columns in proportion to the depth of their upper-end points. The derivation of the curve for the lower catenaries from the higher one by means of the foregoing system of translation is illustrated in Fig. 27. The slopes of the two catenaries, R and R', are equal. Consequently, the slope of the reaction at R' is obtained by drawing the tangent to the catenary, $S\ Q$, at the point, R. In this way the direction of the reaction at any point of the horizontal section under consideration may be determined.

Information concerning the distribution of vertical normal stresses on the horizontal section is lacking. The foregoing graphic procedure permits a

designer to avoid the more cumbersome algebraic operations. The directions of the reactions, P_1, are now known and since the major principal stresses are assumed to be equal to a given f_c, the magnitude of the reactions, P_1, also, is known.

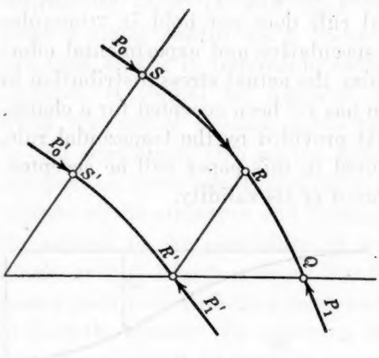

FIG. 27

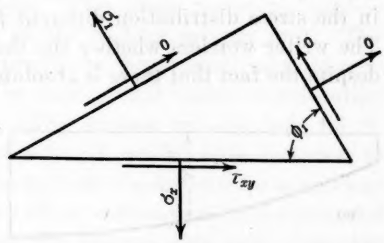

FIG. 28

The magnitudes of the vertical normal stresses at different points of the horizontal section may be evaluated with the assistance of Fig. 28, which represents the equilibrium of the stresses acting on an elementary prism. The catenaries are assumed to represent special trajectories without minor principal stresses. This means that the major principal stress is the only one acting on the two smaller sides of the prism, provided the smaller sides are parts of trajectories. In Fig. 28, note the vertical normal stress, σ_x, and the horizontal shearing stress, τ_{xy}. From the condition of equilibrium on the prism, the following equations may be written:

$$\sigma_x = f_c \sin \phi_1 \quad \ldots\ldots\ldots\ldots (49)$$

and,

$$\tau_{xy} = f_c \sin \phi_1 \cos \phi_1 = \frac{1}{2} f_c \sin 2\, \phi_1 \quad \ldots\ldots\ldots\ldots (50)$$

These values may be determined, since the angles, ϕ_1, at any point are known. They are plotted for illustration in Fig. 29(*a*) and Fig. 29(*b*).

The corresponding curves referring to the buttress with a vertical up-stream face are particularly interesting. The tangent to all catenaries at the left end point of the horizontal sections will be horizontal. The value of ϕ_1 in Equations (49) and (50) is equal to 0. Consequently, both the vertical normal and the horizontal shearing stress will be equal to zero at the left end points of all horizontal sections, as indicated in the curve, Fig. 30(*a*) and Fig. 30(*b*).

It will be noticed that the stress distribution does not follow the trapezoidal rule. The diagrams are peculiar especially in the case of the vertical up-stream face. Engineers are inclined to accept the trapezoidal rule as a matter of fact, and they do not like to discard it unless its fallacy has been proved. Mr. Schorer, however, does not supply this deficiency, since he fails to prove that there is no shear and no minor principal stress along the

catenaries. In fact, there are both normal and tangential loads acting on the sides of the five columns in Fig. 11. Furthermore, it is quite probable that the effect of the foundation is to create a still greater discrepancy between the actual stress distribution and that shown in Figs. 29 and 30, in which the buttress alone is considered. It is universally recognized that because of the influence of the foundation the trapezoidal rule does not hold in triangular dams. It is also known that considerable speculative and experimental effort has been directed toward an effort to determine the actual stress distribution in such a profile.[17] Nevertheless, no suggestion has yet been accepted for a change in the stress distribution different from that provided by the trapezoidal rule. The writer wonders whether the theory offered in this paper will be accepted, despite the fact that there is absolutely no proof of its validity.

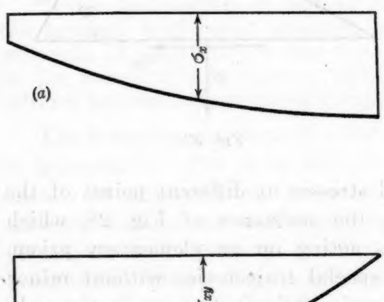

FIG. 29

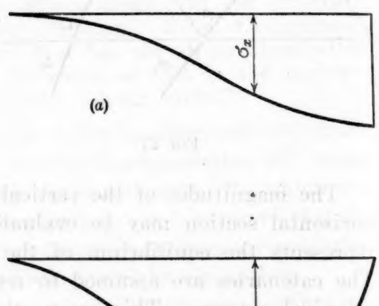

FIG. 30

The author states that his direct procedure makes it possible for a designer to proportion the entire buttress so that it will have a uniform strength; but engineers like to check the structures after they have been proportioned. How should one proceed to check the structure in this case? Should the buttress be treated as a column when it is monolithic? Decidedly, it is not a column. Should it be treated as a beam? If so, the trapezoidal rule should be applied to the individual sections, and the author is opposed to the application of this rule. The buttress units might be treated as laminæ approximately, but their complicated shapes introduce insurmountable difficulties to such treatment. It would be valuable if Mr. Schorer would suggest some way of checking the dimensions selected.

Design Example.—The design example illustrated in Fig. 11 contains nothing revolutionary. Its characteristic features are the subdivision of the buttress into five parts, a somewhat strange "neck", and a decrease in thickness from the up-stream face toward the down-stream face. It contains no sections of the shape indicated in Fig. 27. Of all the innovations only the back por-

[17] "Stresses in Gravity Dams by Principle of Least Work," by B. F. Jakobsen, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., September, 1930, Papers and Discussions, p. 1613.

tion is new, which, by the way, is not a convincing innovation. A perfectly triangular profile of the same volume could supplant it without the least appreciable change in the statical conditions.

There are two construction features that should be discussed. The continuous plate, or slab, that covers the buttresses on the down-stream side is quite useful as a means of preventing buckling. However, its presence makes it impossible to inspect the buttresses and, in the writer's opinion, it should be removed entirely, or, at least, it should be confined to certain isolated parts of the dam. The same reasoning applies to the continuous slab at the inside face of the highest column unit. The prospect of removing these slabs directs attention to the general resisting conditions of the structure and especially to the column unit under consideration as applied to the possibility of buckling. The column unit is only 1.5 ft. thick, and its length is about 300 ft. Such a length would cause designers to worry even with stiffening cross-walls and even if the thickness were as much as 8 ft. at the bottom. By removing the cross-walls at alternate cells, the material thus saved could be used in proportioning the column unit with somewhat greater thickness. In the construction of dams, certainly, there is no room for quibbling over the addition or subtraction of every inch of concrete in place.

Another construction feature may be mentioned in connection with Section *X-X* of Fig. 11. Fig. 11(*i*) shows that the column units change abruptly in thickness from one to the other, while Fig. 11(*d*) shows that the horizontal section at the base has a uniform width. No section is given to show where the difference between Fig. 11(*d*) and Fig. 11(*i*) becomes compensated. The writer does not approve the arrangement in Fig. 11(*i*), Section *X-X*. From a purely theoretical point of view, abrupt changes are not justifiable, because any two hypothetical elementary column units in any of the five columns are subjected to different loads and, therefore, require different cross-sections. From a practical point of view, abrupt changes in thickness do more harm than good.

Conclusions.—A continuous rear slab is not a desirable feature in this kind of design. Furthermore, the shape of the "neck" at the level of 250 ft. is not essential and the abrupt change in thickness at the joints is a poor feature. The writer does not favor the introduction of joints in the design of recent dams for two reasons: First, they weaken the buttress from a structural point of view without making the statical conditions any clearer. Indeed, they make the conditions of static so complicated that they cannot be comprehended at all. The second reason for distrusting the use of joints is that they do not improve the conditions in the buttress in view of the hazards caused by shrinkage and temperature changes. A further remark may be added concerning the buttresses. They should not be made too slender near the down-stream face where the relatively great height of the column unit is most conducive to buckling.

As for the theoretical considerations in the paper, the author does not furnish any proof as to the validity of his theory for the monolithic buttress,

or even for a definite number of columns resting on each other. The writer has established the fact that in monolithic buttresses the author's theory involves proving that the rule of the trapezoid is not valid. For this reason, the proposed method cannot be used for the final selection of the shape of the buttress. It is an excellent way to estimate, without much preliminary effort, an approximate shape for the buttress. It will be necessary, however, to analyze a certain number of horizontal sections by means of the trapezoidal rule. Although, admittedly, this rule is not correct it is to be preferred to any other stress distribution until the newer plan has been proved valid. Obviously, the process of analyzing horizontal sections by means of the trapezoidal rule will be extremely laborious by analytical methods, owing to the irregular change in thickness throughout the entire buttress. That is the disadvantage of this procedure in contrast to the alluring possibility of choosing, directly, the shape of the buttress by Mr. Schorer's method. A graphical method, however, can be easily found for the successive investigation of the single horizontal section by adopting and extending the well-known graphical application of the trapezoidal rule to the analysis of the single horizontal section in a gravity profile of general shape.

The author deserves considerable credit for having introduced the idea of elementary columns, even if it cannot be applied to monolithic buttresses. His method undoubtedly permits a direct selection of a primary, approximate buttress shape so that when his method is completed by means of a supplementary graphical investigation, as suggested in the foregoing remarks, it represents a positive progressive step in the design of dams.

A. FLORIS,[18] ESQ. (by letter).[18a]—It is well known that tensile stresses are produced in dams during the cooling period of the concrete. Furthermore, it is generally recognized that in triangular dam profiles near the foundation the stress distribution, due to water pressure and the weight of the dam, does not remain linear. To reduce the shrinkage in concrete dams to any considerable extent is scarcely possible at present, and the problem of determining stresses at the base of the dam produced by water pressure and the weight of the structure presents many mathematical difficulties that have not yet been overcome. For this reason attempts have been made to avoid these difficulties by introducing joints and thus allowing the concrete to contract, without cracking, along predetermined lines. The dam is then analyzed either approximately by considering these joints or by assuming that the structure is monolithic in spite of them.

In the literature on dams the statement has been made repeatedly in recent years that inclined contraction joints should be located along the trajectories of the first principal normal stress caused by the water pressure and the load of the structure. These trajectories, on the other hand, are determined by means of the rule of the trapezoid and by assuming the dam to be monolithic. However, the introduction of such joints complicates the problem

[18] Civ. Engr., Los Angeles, Calif.

[18a] Received by the Secretary, March 3, 1931.

of stress distribution very much, because the boundary loads transmitted to the given column from the adjacent columns formed by the joints are not known.

One can be reasonably confident that in the near future the problem of determining the stress distribution in gravity dam profiles without joints, and which form a single unit with the foundation, will be solved. Then, the stress distribution in various types of monolithic buttresses will also be known within satisfactory limits. However, this cannot be expected in the case of dam profiles with joints.

Mr. Schorer states that from Castigliano's theorem of least work it can be concluded that the stress distribution in monolithic buttresses will be very nearly the same as that produced in the independent columns. Stresses can be found in prismatic bars (one-dimensional) by the use of Castigliano's theorem, but it cannot be applied to the stress distribution in gravity dam profiles (two-dimensional) and certainly not in the type of buttress suggested by Mr. Schorer, which varies in form even in the third dimension (three-dimensional).

The author has not proved that the catenaries of the independent arched columns will coincide with the trajectories of the first principal normal stress in a monolithic buttress. Furthermore, it is not correct to say that the first principal normal stress in a monolithic buttress will be equal to that produced in the structure with independent arched columns; nor is it correct to state that the second principal stress will vanish in a monolithic structure the same as it does in independent arched columns. It is not at all correct to maintain that the influence of shrinkage and temperature changes will be so great that the effect of the principal normal stress (shearing stress) and Poisson's ratio, etc., can be neglected as relatively small.

The effort to discard the rule of the trapezoid in designing dams by introducing the arched column theory is not quite new. In addition to the fact that dams with inclined joints have been built in recent years, other writers before Mr. Schorer have treated the same subject. In studying failures of dams, Paul Ziegler came to the conclusion that a kind of arch action takes place in gravity and buttressed dams.[19] He points out that cracks produced in the remaining buttresses of the Gleno Dam which failed and also in other gravity dams, indicate that arches are probably formed in gravity and buttressed dams under stress. Such arches apparently have a horizontal abutment at the up-stream face while the other is resting either on rock in the lower part of the dam, or is carried in the upper parts of the profile by the lower arches as shown in Fig. 31. The author attempts to locate these arches and thus to determine the stresses in the dam graphically by using funicular polygons.

The method proposed by Fredrik Vogt, Assoc. M. Am. Soc. C. E., is exactly the same as that presented in this paper.[20] By dividing the buttress of a monolithic dam into a number of independent columns subjected to the load,

[19] "Volle Strebenmauer and Pfeiler-Strebenmauer," von Paul Ziegler, *Schweizerische Bauzeitung*, June 29, 1929, p. 315.

[20] "Economical Design of Buttresses for High Dams and of Cellular Gravity Dams", by Fredrik Vogt, Det Kgl. Norske Videnskabers Selskab, Forhandlinger, Bd. II, No. 40, p. 141.

p_0 (see Fig. 32), Dr. Vogt derives the following differential equation for the column axis:

$$\frac{d^2 y}{dx^2} = q \left[1 + \left(\frac{dy}{dx} \right)^2 \right] \ldots\ldots\ldots\ldots (51)$$

in which, $q = \frac{\gamma_w}{\sigma}$, $\gamma_w =$ the water pressure, and $\sigma =$ the allowable compressive stress. Let $y = 0$, and let $\frac{dy}{dx} = \tan \alpha$; when $x = 0$, Equation (51) becomes:

$$y = \frac{1}{q} \log \frac{\cos \alpha_0}{\cos \alpha} \ldots\ldots\ldots\ldots (52)$$

in which, $\alpha = \alpha_0 + q\,x$.

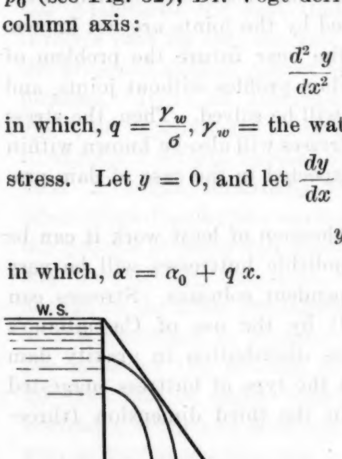

FIG. 31

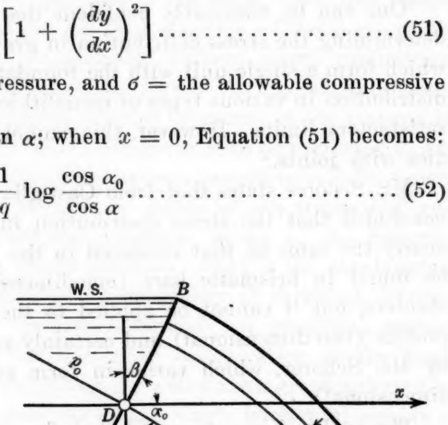

FIG. 32

The curve represented by Equation (52) is, as in the author's case, a catenary. Let the thickness equal,

$$t = \frac{A}{\alpha} \ldots\ldots\ldots\ldots (53)$$

in which, $A = \frac{p}{\sigma} = \frac{p_1}{\sigma \cos \alpha}$, and $p_1 =$ the constant horizontal component of p. By selecting the thickness of the columns properly, they may be designed so as to be subjected to the same compressive stress at every point. Because the width, d, of the columns decreases toward the base of the dam directly in proportion to the increasing pressure, p, to maintain uniform strength throughout, it is clear that the thickness of the column must be increased accordingly. Furthermore, by determining the minimum value for the expression of the sum of deck and buttress weights between two adjacent elevations, the following expression for the variable slope, β, of the up-stream curve, ADB, in Fig. 32, can be derived:

$$c^2 F z^4 + 2(1 + c^2) F^2 z^3 + [(1 + 4c^2 + c^4) F^2 - 1] z^2 + 2cF[(1 + c^2) - 1] z = (F - 1)(1 - c^2 F) \ldots\ldots (54)$$

in which, $z = \sin \beta$, $F = e^{2qh}$, and c varies from 0 to 0.2. The constant, c, expresses the ratio of deck weight to water pressure, and in high multiple-

arch dams it is approximately equal to 0.1. By solving Equation (54), with respect to z, the up-stream curve of the dam can be traced step by step.

The same procedure is followed by Mr. Schorer, and his proposed dam profile (see Fig. 10) is, therefore, necessarily the same as that presented by Dr. Vogt.

FREDRIK VOGT,[21] ASSOC. M. AM. SOC. C. E. (by letter).[21a]—The writer has studied this paper with the greatest interest, particularly because he has worked along the same line himself, and published a short paper on this subject in 1929.[22] It is gratifying to note that the general point of view is the same in both papers, and that even the formulas check each other to a great extent, except for differences in nomenclature.

The writer has not much to add, therefore, except to state that theoretically the most economical shape of buttresses approaches very close to that of a straight-faced dam when the cost of the arches is included in the analysis. Fig. 10 gives this shape for a deck factor, 0, defined by Equation (22), that is, neglecting the cost of the arches. For actual dams the deck factor averages 0.1. Fig. 33, taken from the writer's paper, illustrates the shapes corresponding to deck factors 0, 0.1, and 0.2. It may be noted that a deck factor, 0.1, makes the up-stream face, determined in this way, steeper than if this factor were zero. Furthermore, the up-stream face is almost entirely straight if the deck factor is 0.1, or more. For practical design the writer, therefore, suggested a straight-faced dam as shown by the dotted lines. This shape has also been adopted by the author.

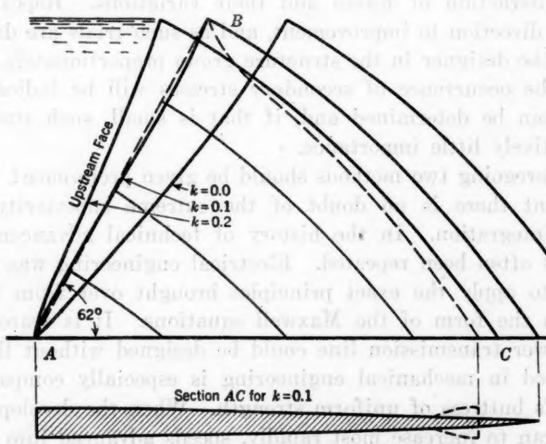

FIG. 33.—MOST ECONOMICAL SHAPE OF BUTTRESSES WITH DECK FACTORS 0, 0.1, AND 0.2, AND BUTTRESS RATIO, $t = 0.5$.

Dams with steep up-stream faces require a minimum amount of concrete, but the sliding factor computed for horizontal joints does not satisfy the usual

[21] Prof., Norges Tekniske Höiskole, Trondhjem, Norway.

[21a] Received by the Secretary, March 16, 1931.

[22] "Economical Design of Buttresses for High Dams and of Cellular Gravity Dams," Det Kgl. Norske Videnskabers Selskab, Trondhjem, Norway, *Proceedings*, 1929, No. 40.

standards. Therefore, the saving in concrete work can only be effected if sliding is prevented by cutting large steps in the rock foundation approximately normal to the lines of thrust for each of the columns into which the buttress is divided. Furthermore, if sliding is to be prevented in this manner, the rock foundation should be perfectly sound.

A. C. SCHWAGER,[23] ESQ. (by letter).[23a]—The strict solution of a problem, that has been sought since the beginning of modern dam design, is presented by the author. The writer proposes to analyze the mathematical parts of this paper without regard to the construction aspect described by Mr. Schorer. The method may be outlined concisely, as follows:

(1) The design specifies that only compression stresses, constant throughout any buttress, shall exist. Therefore, each hypothetical column in a buttress may be assumed to be a unit, and its equilibrium can be investigated independently of adjacent column units in the buttress.

(2) An infinitely small element is selected and the differential equation resulting from its condition of equilibrium is set up.

(3) These differential equations are integrated and are made to coincide with the boundary conditions.

Some engineers regard the foregoing procedure as if it were based entirely on abstract and scientific premises. The idea of stating the problem in terms of infinitesimally small volumes seems especially impractical to them. It appears much more natural to sketch an outline of a buttress, analyzing it and dissecting it in a manner that enables the practical engineer to visualize the action and interaction of forces and their variations. Repeated trial analyses show the direction to improvement, and as such trials are duplicated the confidence of the designer in the structure grows proportionately. At the various sections the occurrence of secondary stresses will be indicated, but their magnitude can be determined and, if that is small, such stresses are considered of relatively little importance.

Which of the foregoing two methods should be given preference? From a scientific standpoint there is no doubt of the extreme superiority of the method of exact integration. In the history of technical advancement the same situation has often been repeated. Electrical engineering was the first technical science to apply the exact principles brought over from the field of pure physics in the form of the Maxwell equations. It is scarcely conceivable that a power transmission line could be designed without them. A problem encountered in mechanical engineering is especially comparable to that of the author's buttress of uniform strength. When the development of steam turbines began to increase most rapidly, speeds advanced into a range previously unknown. As a consequence, the stresses in the disk wheels produced by their own centrifugal forces represented the major design problem.

Trial methods based on successive graphical solutions were inaccurate and time-consuming. A small proportion of the total weight, placed incorrectly, caused inefficiency and resulted in designs that were dangerous to life. It

[23] Oil Circuit-Breaker Design Engr., Pacific Elec. Mfg. Corporation, San Francisco, Calif.

[23a] Received by the Secretary, April 1, 1931.

was De Laval and Stodola who introduced the requirement of constant radial and tangential stresses. They integrated the differential equations and obtained the disk of uniform strength. The resulting formulas have made it possible to eliminate all guesswork. They are considered definitely correct and have been accepted for several decades as the tools of every turbine designer.

In the case of the buttressed dam of uniform strength the same arguments apply as in that of the transmission line or the turbine wheel. The scientific analysis offers the only correct solution eliminating waste of materials and time. The success of Mr. Schorer's method of solution from a technical standpoint cannot be doubted, but it is just as important from a scientific standpoint, since once more it has proved the tremendous power that is available in this use of differential and integral mathematics. It is not possible to over-emphasize the fundamental importance of stating a problem in differential equations wherever possible as set up by the exact science, and of finding the integral effects applicable to the particular condition.

Mr. Schorer deserves credit for applying this method to an engineering problem of major importance and for obtaining such an elegant solution of a buttressed dam of uniform strength.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

HIGHWAY LOCATION: PRACTICAL CONSIDERATIONS

Discussion

BY MESSRS. J. C. CARPENTER, H. J. SPELMAN, H. S. KERR, AND H. W. GIFFIN

J. C. CARPENTER,[3] M. AM. SOC. C. E. (by letter).[3a]—The paper by Mr. Losh brings out the fact that highway location in the past has followed the path of least resistance. This is obvious to any one who studies the existing system. Until a broader policy is adopted by the officials in control of this important feature of engineering the same mistakes will be made as are now evident in past practice.

The first State highways were located adjacent to the larger centers of population and the location was not a matter of serious thought, the object being to improve the "old road". With the passage of the Federal Aid Road Act, in 1916, the States were required to map out a system of highways, on which Federal Aid could be expended. The systems thus laid out were not limited to mileage, and while an attempt was made to provide a system of transportation that would properly serve the State, the result was not at all satisfactory. The projects built on this system were more or less development roads and some of them are now being eliminated as parts of the system. Short sections, radiating in all directions from the centers of population, were the rule, rather than the exception, under the procedure in vogue at that period of the highway development program.

The "Federal Highway Act" of 1921 added some desirable features which were introduced with the intention of developing a system of properly located, connected routes. A new system of primary and secondary highways was adopted, and the mileage was limited to 7% of the total public highway mileage of the State. This system was approved by routes with official control points tied in to a description. The procedure in approving projects requires that the routing between official control points be determined and approved before any part of the route is improved. This was undoubtedly a step forward and has served to locate the improvements on a system of main high-

NOTE.—This paper by A. R. Losh, Assoc. M. Am. Soc. C. E., was presented at the Joint Meeting of the Highway and Construction Divisions, Dallas, Tex., April 25, 1929, and published in November, 1930, *Proceedings*. Discussion on the paper has appeared in *Proceedings* as follows: February, 1931, by H. K. Bishop, M. Am. Soc. C. E.

[3] Fort Worth, Tex.

[3a] Received by the Secretary, February 13, 1931.

ways, "interstate in character". There have been numerous instances, however, of poor locations due to the necessity of tying in the old work that was completed under the first system. The plan also results in "center-to-center" locations which lead the traffic to the congested parts of the larger centers of population.

Under the foregoing procedure the highways of the United States have been developed in such a way that it is possible to travel to all sections on some type of improved road. Traffic trends are becoming stabilized. It is an opportune time to "take stock" and do some real planning that will serve to provide some permanent locations. With the present systems as a basis, a revised system of highways can be laid out in such a manner as better to serve local and through traffic for the entire country. It is important that such a system be developed at an early date. Based on present and probable traffic trends, on the development of the industrial and agricultural sections of the country, the growth of cities, and the controlling topographic features, a careful study should be made in each State and a major system laid out and filed for use on new work. It will be necessary carefully to consider entirely new locations, extending across the State, in some instances. The principles discussed in the paper should be applied in developing this ideal system, as, for instance, the development of locations through cities, locations at suitable distances from railroads, etc. While it will be impossible to change from the present system to such an ideal system at once, the data should be compiled and the system developed and placed on file with the expectation of utilizing it as funds become available. With such a plan in reserve, a State Highway Department is prepared to justify a change from the present routing and any short section constructed on the major plan eventually will become a part of the larger plan. Under the present policy, large outlays of money are required to reconstruct old projects on narrow rights of way with undesirable alignment, and traffic congestion is increased rather than diminished. With the same funds a new highway can often be built on good alignment, with adequate width of right of way for future development and with much better planned surfacing, leaving the old road to serve local traffic with a much smaller annual maintenance cost for the two routes. The new location develops and serves an additional local community and, in addition, provides ample room, at low ground rental, for commercial activities necessary for highway traffic, such as garages and hotels, without interfering with established stores and other services, essential to the community, on the old road.

In determining the location through small centers it is sometimes advisable to route short sections of the highway on an undesirable location, as a temporary measure. The location should be so developed outside the village that, when traffic increases to a point such that the local congestion becomes unbearable, the road can be projected past the town on a direct location, leaving a detour through the village. All highway departments are more or less dependent on local sentiment for their popularity. They cannot carry out a large program, even where purely State funds are available, unless they have the active support of the people. Opposition of a small community may be serious. In the early development of the highway system for any locality,

due to the small amount of through traffic, there is no very serious operation expense involved in routing the highway along a longer line to carry it through the center of trade for the unit under consideration. If the line is on a through highway the foreign traffic will increase in a short time, to such an extent that the same interests that were insistent upon the "Main Street" location will be clamoring to have this outside traffic taken off their city streets to save maintenance expense and relieve the traffic congestion. Local people will then "discover" the short direct line outside the city and, as has sometimes happened, they may provide the funds to build the by-pass. If the traffic does not develop in a measure that will induce this action, the road is not of enough importance to be classed as a through highway. Local traffic will still be the portion to be accommodated and the line should follow the main street.

H. J. SPELMAN,[4] M. AM. SOC. C. E. (by letter).[4a]—Unquestionably, too many of the roads built during the life of the present generation have become obsolete too early; long before the expiration of the "reasonable life" anticipated by the engineer who located and designed them. In the future, highway engineers must locate more boldly than ever, lest the next generation find locations of the present day as obsolete as those built in 1915 or 1920 have become in 1931.

In 1919, and prior to that time, the average motor vehicle traveled at a speed of 20 to 25 miles per hour. The average motor vehicle now travels at a speed of from 40 to 50 miles per hour. Who knows but that the average speed of a generation from now, on many roads, may not be from 60 to 75 miles per hour, with 100 and 125-mile speeds at least as common as the 65 and 70-mile speeds at present? It is the increases in speeds that have caused the older road locations to become obsolete, because these higher speeds require gentler curvatures, longer sight distances, and greater widths of paving, roadway, and right of way.

In making present-day locations, highway engineers should not only locate with a view to traffic conditions as they are to-day, but should endeavor to anticipate those of the future. The future will undoubtedly bring higher speeds which will create a need for still wider sections and gentler curvatures.

As Mr. Losh has stated, property values increase very rapidly adjacent to a road after it is improved, and, hence, re-location and widening of a road once built, are particularly expensive. This again stresses the need for location and width of right of way adequate for the future. It is very much a question if any highway department to-day is obtaining rights of way of adequate width for the future on very many of the roads they are building.

In many cases a road has been located under low standards of alignment, and with narrow section and right of way, on the theory that such a road was a secondary or tertiary road that would never carry much traffic. With the improvement of the road, traffic flowed to it, and it really became a primary road from a traffic standpoint. Consequently, it soon become inadequate and

[4] Washington, D. C.

[4a] Received by the Secretary, February 27, 1931.

required reconstruction. In order to avoid such occurrences the only remedy would appear to be a study of the traffic needs of the entire community or district, and a comprehensive road plan for such community or district, in the same way that city planning is now being done in many of the larger urban centers.

This paper seems particularly valuable in that it is an excellent summary of just what the highway engineer-executive must consider in making highway locations. It is valuable, too, in pointing out the mistakes of location in the past; and it is to be hoped that a study of these mistakes will enable the avoidance of similar ones in future locations.

H. S. Kerr,[5] M. Am. Soc. C. E. (by letter).[5a]—The major problems of highway location as presented in Mr. Losh's paper have been particularly well handled. The writer is generally in accord with the views therein set forth and can consistently approve the conclusions reached.

The prime consideration in highway location, with reference to topographic conditions, is, of course, to find the man who is a competent locator. In all the State highway departments the country over there are an extremely limited number of men gifted with that "sixth sense" which is the peculiar distinction of the high-class locator, and when such men are found, their worth is beyond price. In the old days of active railroad expansion these "sixth sense" locators were always in great demand, even if in some instances they might be decidedly below the average in handling engineering work aside from locating. The policy of the State of Utah is to confine difficult locations to the few men in the State Highway organization who have this qualification, and to apply railway methods. No better method has been evolved than an adherence to the long established railway practice of projecting the location in difficult sections, based on a carefully determined preliminary line, established after thorough consideration has been given to all alternate routes possible. Aerial photography is a help in finding the satisfactory location, and it is to be regretted that this method was not available several years before the problems (at any rate, as affecting interstate highways in Utah) were practically solved. Even though aerial photography came into vogue after the difficult locations in Utah were made, the final results are eminently satisfactory. The writer knows of no revisions that merit consideration in the case of any highway locations in this State during the eight years, 1924 to 1931, inclusive.

While agreeing that short sections of road with grades as steep as 8% are, in some instances, allowable, there are only a few such cases in Utah, although it is a mountainous State, where such gradient applies. In these cases a future reduction to 6%, in most cases without re-alignment, is reasonably practicable when traffic increase justifies the change.

In Utah the counties and the Road Commission have the power to protect the right of way from encroachments and prohibit advertising signs thereon, and there is necessity for constantly combatting the tendency, noted in

[5] Chf. Engr., State Road Comm., Salt Lake City, Utah.

[5a] Received by the Secretary, March 13, 1931.

the paper, of infringing upon the public right in this respect. A number of earlier projects, including some introduced subsequent to the enactment of the Federal Aid Act, have a right-of-way width of 66 ft. which it is proposed to increase, where feasible, to 100 ft., the general standard. Even wider rights of way are now being considered for roads across the public land areas.

With reference to the conflicting interests between local and through traffic, the conditions cited are those that have formerly obtained in Utah, and, in many instances, it would not have been practicable to have attempted to work out an ideal location, particularly between 1915 and 1925, during which period local requirements were largely controlling factors in location. However, now that the towns and cities have generally been taken care of, to the maximum extent possible, between control points, consideration may properly be given to the best location for accommodating through traffic, utilizing to this end, portions of existing secondary highways, in by-passing congested areas and in reducing mileage. This would not have been practicable between 1915 and 1925 because the need of good roads was then so urgent in connecting centers of population. There are only a few cases in Utah, however, where such revisions are necessary on this account, or because construction standards are so much higher now (1931); this latter consideration is due to the fact that the first requirements in Utah were for roads in the valley sections where construction costs were relatively light and grading and structure costs per mile were not such as to cause public opinion to interfere with suitable location. In later years highway improvements have been extended from the more populous districts, to and through the mountain and desert sections, to the State borders. In so doing it has been possible to finance projects which average, in some instances, far in excess of $30 000 per mile, an accomplishment which would not have been feasible some years earlier. Now, however, it can be readily effected on account of the more recent trend of public opinion with reference to highway costs. The result has been that location involving heavy work during several years past has been entirely satisfactory.

With reference to location the following factors should be given due consideration, namely, annual interest, maintenance, and operation costs. Only the first item can be computed accurately, the remaining two, at best, can be but closely approximated, the result depending upon the ability and judgment of the locator in properly considering all items appertaining to each and every project.

Railroad grade crossings have been and are being eliminated by the Utah State Highway Department as rapidly as possible, the majority by relocation rather than by structures. In regard to co-operative railroad costs, the practice in Utah has been, unless conditions justified a variation, to divide the costs of structures on a "50-50" basis, and to arrange for the most satisfactory highway alignment, resulting generally in skew crossings. The most recent grade separation divided the costs equally for the overhead structure and the approach fills. Elimination of grade crossings by revision, in the case of low-cost construction, is handled on a "50-50" basis also, but where heavy work is involved, a contribution on the part of the railroad benefited,

is arranged on the basis of $9 000 per double-track, main-line crossing and $6 000 for a single-track crossing. The construction or the vacating of railroad crossings is, in Utah, subject to the approval of the Public Utilities Commission. However, with respect to grade crossings on State highways, the Commission requests that the Highway Department present, in advance of hearing, the details and stipulations tentatively agreed upon by that Department and the utility concerned.

Improvements are under way in the State on U. S. Highway No. 50. From a connection with U. S. Highway No. 91 this route extends across the mountains and the desert a distance of 223 miles, on the original route, to the Colorado line. The problems of location are, in the main, purely of an engineering character; they are entirely so from Green River eastward to the State line, a distance of 85 miles. The original route included twenty-eight railroad grade crossings, numerous right-angle turns, steep gradients, and sharp curves. Elevations varied from 4 080 to 7 450 ft. In the re-location survey of this highway it was possible to obtain results eminently satisfactory from an engineering standpoint, there being only a few restrictions applying that prevented an ideal location. The re-alignment effects a saving on the total distance of 14 miles. The major portion is now (1931) completed or under construction. All double-track crossings will have been eliminated by the programmed construction. A few single-track and relatively unimportant crossings will be temporarily continued although the plans call for the ultimate elimination of these crossings also. Little value is attached to the old highway between Price and Colorado, 155 miles, which lacked even numerous necessary bridges. The new 223-mile line, when entirely completed, will have less than one-half the total degrees of curvature of the trans-continental railway which it approximately parallels. Minimum surfacing requirements call for 18-ft. gravel surfacing of 6-in. depth, in two courses, 100% of which must pass a 1-in. screen, round opening. From this, types range to 12 miles of completed concrete pavement.

The total cost, including 1931 construction, is estimated at $3 047 548, with an estimate of $838 000 additional, to complete, or an average cost of $18 800 per mile. Considerably less than this average develops on the desert section of Eastern Utah, while on the mountain section, costs, in 1930, on some projects ran as high as $60 000 per mile for grading and structures only Of the gravel mileage about one-half will be oil-treated within a short time and some miscellaneous paving will be done, which costs are not included in the foregoing figures.

H. W. Giffin,[6] Assoc. M. Am. Soc. C. E. (by letter).[6a]—Highway location is not altogether an engineering problem. The construction of highways is dependent on public support and the locating engineer or highway planner must constantly keep in mind that his handiwork must be approved by the present generation, irrespective of how future generations may regard it. Perhaps in the future the highways of to-day will be judged as the highways

[6] Field Engr., State Highway Dept., Trenton, N. J.

[6a] Received by the Secretary, March 30, 1931.

of a former day are now regarded. The principal reason for highway construction to-day is the love of the human race for swift, smooth motion without effort. Probably no highway bond issue was ever floated because of a desire to increase the economy of transportation. The policies of the highway planner in determining location are the result of his estimate of public approval and it is useless to try to get too far ahead of that. This means some waste as the years go by and locations become obsolete with changing conditions and the greater demand for locations approaching more closely to the ideal.

The author states that "curves should be placed where they may be seen readily by the approaching driver". It would be well to go further and say that every condition of the highway, whether it is a change of direction, an intersection, a railroad crossing, or anything that requires the driver's attention, should be placed, or otherwise brought to his notice, so as to give him time to take appropriate action. It is suggested that some advance designation, such as a number denoting the sharpness of the curve be used on signs which now denote curves and their direction. Although some motorists would not know what a stated degree of curve meant, they would soon learn to associate the numbers with the relative sharpness of the curves. To-day, the same sign that frequently is used to warn the motorist of a sharp curve is also used for flat ones.

Because of the adoption of some degree of curve as a maximum, locating surveyors do not always feel that every effort should be made to secure flatter curves. Long flat curves mean additional work for the survey party, but such curves should be as flat as 30′, cost permitting. Where necessary to have a curve near a summit it should be located if possible so as to extend for some distance on both sides. Where vertical curves are coincident with horizontal curves on summits, pleasing alignment is attained. It is not easy to define the conditions which produce an alignment that displeases the eye, but it is suggested that the location that contains, within one view, three tangents and two curves, or two complete changes of direction, whether horizontal or vertical, is not pleasing.

The sight distance desired along the highway is governed by the prevailing speed of the traffic at the particular location. Highways laid out for fast traffic (that is, those which are wide and reasonably straight) in the country should have sight distances of 800 to 1 000 ft. if obtainable at reasonable cost. City conditions do not require more than 300 ft. Where difficulty is experienced in obtaining satisfactory sight distances the roadway, under some circumstances, may be separated into one-way roads, thus permitting the required sight distance to be cut in half.

Because of the relative high cost per unit of length in comparison with the remainder of the highway, bridges have exerted a great influence on location. Despite this high cost, the location engineer should remember that the ordinary bridge is only a part of a highway, and is often unnoticed by the motorist unless the alignment impresses it upon his mind. An otherwise satisfactory alignment is sometimes spoiled because the requirement of securing a less costly bridge is exaggerated.

The segregation of through and local traffic is desirable and advantageous to both classes, but it is seldom reasonable or economical until each reaches sufficient volume to justify separate roadways. The construction of the by-pass is the common method of accomplishing this purpose. This may be termed second-stage planning and has not been as readily understood and approved by the public as the first stage, which may be defined as the extension of the main street of one town to that of another.

Near metropolitan centers the subject of intersections is an important, and often neglected, phase of highway location. The capacity of a highway is limited by its intersections and, obviously, the combined capacity of two intersecting highways is limited to that of the larger unless some special provision is made to increase the intersection capacity. Where new highways are planned, that cross a network of other important highways, it is sometimes necessary for the locating engineer to study first the available locations for the type of intersection suitable to the requirements.

It is not necessary to go far back in one's memory to prevailing speeds of 25 to 30 miles per hour. Prevailing speeds to-day are at least 20 miles per hour faster. Must the highway planner anticipate speeds of 70 miles per hour as the ordinary condition 10 or 15 years hence?

While it seems wise in laying out new highways to obtain sufficient right of way for widening within a few years, too much planning for the future should not be attempted. How many could have appreciated to-day's highway problems ten or fifteen years ago? Any attempt, other than widening, to plan for to-day's highways fifteen years ago would probably have been unappreciated at that time and would probably prove unsatisfactory now.

Mr. Losh's excellent paper has focused attention on some important problems of highway location.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

FLOW OF WATER IN TIDAL CANALS

Discussion

By H. F. Flynn, M. Am. Soc. C. E.

H. F. Flynn,[39] M. Am. Soc. C. E. (by letter).[39a]—In that portion of the paper dealing with the Chesapeake and Delaware Canal, a comparison is made between observation and prediction, but only one day's heights and times and no currents were then available. The purpose of this discussion is to make such a comparison based on reasonably long series of observations. The material that will be used and the derived tidal constants are shown in Table 9.

The elevations in Table 9 refer to the United States Engineer Department datum in the Delaware River. The locations of the tidal stations are as follows: (1) Delaware City, on the west bank of the Delaware River, 1.8 miles above Reedy Point; (2) Reedy Island, on Reedy Island on the west bank of the main channel of the Delaware River, 3.3 miles below Reedy Point; (3) Reedy Point, just inside the outer end of the north jetty at the Delaware River end of the canal; (4) Biddle Point, on the north bank of the canal, 3.1 miles from Reedy Point; (5) Summit, on the bank of the canal at Summit Bridge, 9.2 miles from Reedy Point; (6) and (7) Chesapeake City, on the bank of Back Creek, at the entrance to the canal, 14.0 miles from Reedy Point; (8) Court House Point, on the southeast bank of Elk River, about 1 mile below the mouth of Back Creek and 5 miles from Chesapeake City; (9) Town Point, on the southeast bank of Elk River, about 1.5 miles below Court House Point; (10) Reybolds Wharf, on the southeast bank of Elk River, about 4.5 miles below Town Point; and (11) Frenchtown, on the southeast bank of Elk River, about 5 miles above Court House Point and about 4 miles above the mouth of Back Creek.

Before proceeding with comparisons between the observed and predicted tides in the canal proper, it will be well to look into conditions at the entrances before and after the canal was opened to tidal flow. No preliminary observations were made right at Reedy Point, the nearest being those at

Note.—The paper by Earl I. Brown, M. Am. Soc. C. E., was published in December, 1930, *Proceedings*. Discussion of the paper has appeared in *Proceedings* as follows: February, 1931, by R. L. Faris, M. Am. Soc. C. E.; and March, 1931, by Messrs. G. T. Rude, Eugene E. Halmos, and W. M. Black.

[39] Engr., U. S. Engr. Office, Philadelphia, Pa.

[39a] Received by the Secretary, March 11, 1931.

Delaware City, in 1923, and those at Reedy Island. Interpolating between these values, the resulting lunitidal intervals at Reedy Point are 11.10 hours for high water and 5.76 hours for low water. Comparing these values with those in Table 9, it appears possible that the duration of rising tide may have become somewhat greater than before, which is to be expected. The elevations of high and low water observed at Reedy Point are normal for this part of the Delaware River, but those at Delaware City in 1923 appear to be somewhat low.

TABLE 9.—TIDAL DATA IN CHESAPEAKE AND DELAWARE CANAL, AND VICINITY

No.	Station	Year	Series	Lunitidal Intervals, in Hours		Elevations, in Feet		Mean tide level, in feet	Range, in feet
				High water	Low water	High water	Low water		
1	Delaware City............	1923	4 months	11.27	5.93	5.97	0.47	3.22	5.50
2	Reedy Island..............	1929	12 "	10.78	5.45	6.20	0.63	3.42	5.57
3	Reedy Point..............	1928–29	9 "	11.19	5.59	6.16	0.79	3.48	5.37
4	Biddle Point..............	1928–29	8 "	11.46	5.90	6.10	1.01	3.56	5.09
5	Summit,..................	1928–29	9 "	11.18	4.56	5.30	1.93	3.62	3.37
6	Chesapeake City..........	1923	5 "	9.28	3.32	4.47	2.15	3.31	2.32
7	Chesapeake City..........	1928–29	9 "	9.81	3.64	4.53	2.23	3.38	2.30
8	Court House Point........	1929	7 "	9.04	3.09	4.32	2.10	3.21	2.22
9	Town Point................	1898–1927	26 days	8.97	3.01	....	....	....	2.42
10	Reybolds Wharf...........	1898–1927	58 "	8.79	2.55	....	....	....	1.93
11	Frenchtown...............	1899	14 "	8.87	2.93	....	....	....	2.43

At Chesapeake City conditions are very different. High water occurs 0.53 hours, and low water 0.32 hours, later than formerly. There are three stations in Elk River for which tidal constants are published by the U. S. Coast and Geodetic Survey; namely, Town Point, Reybolds Wharf, and Frenchtown. At each of these places one or more short series of observations were made between 1898 and 1927. The means of all results for each station are given in Table 9. Comparing the lunitidal intervals from seven months' observations at Court House Point in 1929 with the intervals at the neighboring stations, it is apparent that there has been no material change in the times of tide in Elk River since the canal was opened. There appears to have been no material change in tidal heights in Back Creek since the opening of the canal.

In Table 10 are given in parallel columns the observed and predicted times and heights of tide at the four canal stations. Times are referred to high water at Reedy Point, and heights, to mean tide level at the same point, 3.48 ft. above the datum.

The predicted values are in fair agreement with the observed values, following them in a general way, and are satisfactory for a preliminary computation. It is believed, however, that the method developed by the author is capable of bringing out closer results. In computing the predicted values, it was necessary to make certain general assumptions as to the conditions that

TABLE 10.—COMPARISON OF OBSERVED AND PREDICTED TIDES IN CHESAPEAKE AND DELAWARE CANAL

Station	High Tide				Low Tide				Duration, in Hours				Range, in feet		Mean tide level, in feet	
	Time, in hours		Height, in feet		Time, in hours		Height, in feet		Rise		Fall					
	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted	Observed	Predicted
Reedy Point	0.00	0.0	+2.68	+2.90	6.82	6.2	−2.69	−2.89	5.60	6.2	6.82	6.2	5.37	5.79	0.00	0.00
Biddle Point	0.27	0.0	+2.62	+2.97	7.13	6.2	−2.47	−2.26	5.56	6.2	6.86	6.2	5.09	5.23	+0.08	+0.35
Summit	−0.01	−0.5	+1.82	+1.75	5.79	5.5	−1.55	−1.12	6.62	6.4	5.80	6.0	3.37	2.87	+0.14	+0.31
Chesapeake City	−1.38	−1.9	+1.05	+1.18	4.87	4.2	−1.25	−1.15	6.17	6.3	6.25	6.1	2.30	2.33	−0.10	−0.02

would prevail in the canal. An examination of the discrepancies brought out by a comparison of the observed and predicted results leads to the belief that they can be reduced materially by a revision of the preliminary assumptions.

The computations are based on a canal 14 miles long, the length of the excavated portion. The eastern end of the canal connects with the broad estuary of the Delaware River, and such changes in the tidal regimen as may have occurred there are naturally small. At the western end, the connection is with Back Creek, a small tributary of the Elk River. The distance from the Chesapeake City end of the canal through Back Creek to the waters of Elk River is about 4 miles. Back Creek has a width of about 350 ft. at Chesapeake City, gradually increasing to about 1 700 ft. at its mouth; it has a mean depth of about 7 ft. at mean tide level. These dimensions show that this waterway, especially in its upper portion, is of the same order of magnitude as the canal itself. The Elk River, however, is far wider and deeper, forming a body of water so large that it can be expected to maintain its tidal regimen without material change from the comparatively small affect of the canal waters. That this expectation is reasonable is confirmed by the discussion of the changes in tidal times in the vicinity, given previously.

It seems clear that better results can be secured by computing the predictions for a canal 18 miles long than for one of 14 miles, even though this introduces some complications, due to the modifications necessary to take care of the continually changing dimensions in the 4 miles of channel lying in Back Creek.

It is believed preferable to use the observed differences in time of high tide and low tide at the two ends of the canal separately rather than a round value for both. Judging from the lunitidal intervals at Chesapeake City in 1923, and those at Court House Point in 1929, the values at the mouth of Back Creek probably would have been approximately 9.1 hours for high water and 3.1 hours for low water. The probable values at Reedy Point, as shown previously, were 11.1 hours and 5.8 hours, respectively. The differences for the ends of the 18-mile canal are, therefore, 2.0 hours for high water and 2.7 hours for low water.

In the Delaware River, in the vicinity of Reedy Point, the duration of rise of tide is about 1.7 hours less than the duration of fall; but in Elk River, at Court House Point, the difference is only 0.5 hours. In the computed predictions the durations of rise and fall are assumed to be equal at Reedy Point.

For a comparison between observed and predicted velocities in the canal, there are five sets of simultaneous observations at each of the four canal stations. Each set extends over an entire tidal cycle. In Table 11 the mean values of the westerly and easterly velocities, observed and predicted, are shown in parallel columns for each of the canal stations.

The double predictions at Biddle Point (Items 2 and 3, Column (4)) refer to the sudden change in velocity due to the change in canal cross-section. The values in Item 2 pertain to currents between the station and Reedy Point and those in Item 3 to currents between the station and Summit.

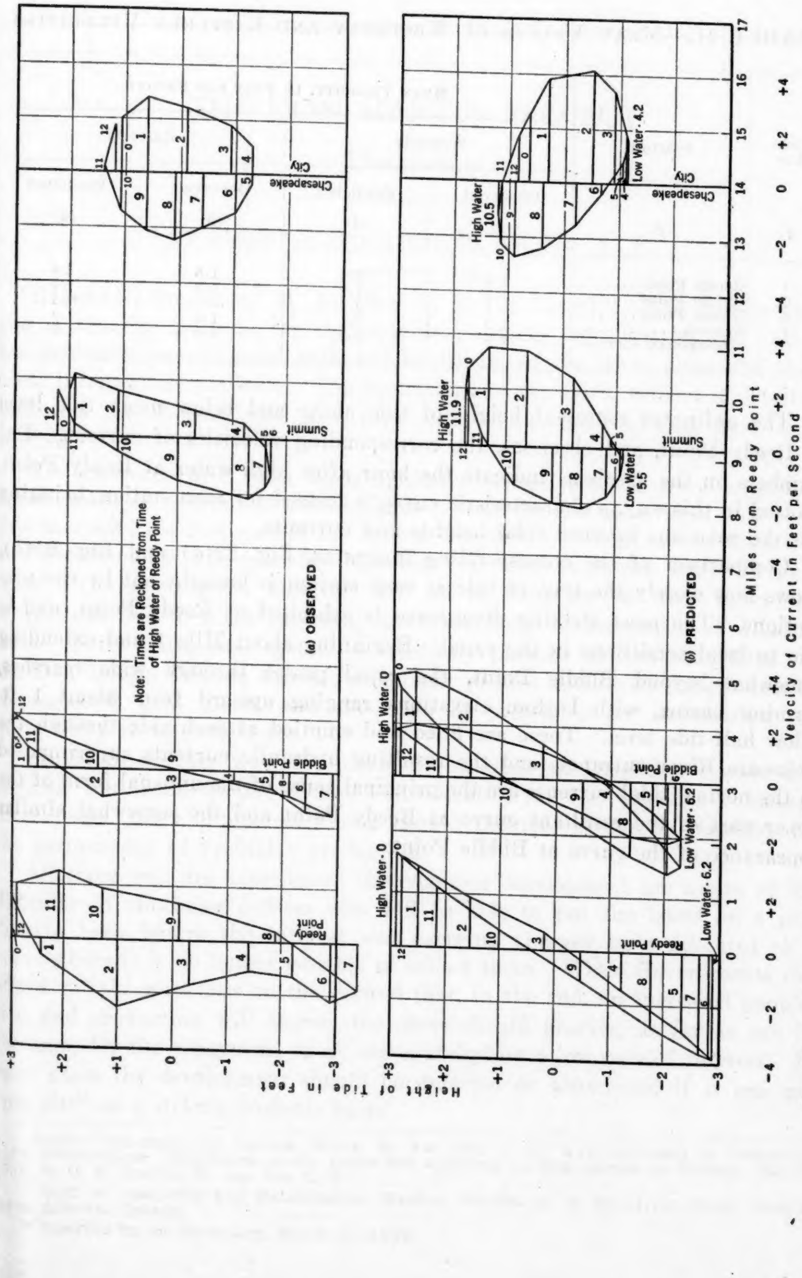

FIG. 22.—HEIGHTS OF TIDE AND VELOCITIES OF CURRENT FOR EACH HOUR (0 to 12), CHESAPEAKE AND DELAWARE CANAL.

The relationship of tides and currents in the canal is shown in Fig. 22. Fig. 22(*a*) exhibits the observed tides and currents; and Fig. 22(*b*), the predicted tides and currents.

TABLE 11.—MEAN VALUES OF WESTERLY AND EASTERLY VELOCITIES

Item No.	Station	Mean Velocity, in Feet per Second			
		Westerly		Easterly	
		Observed	Predicted	Observed	Predicted
(1)	(2)	(3)	(4)	(5)	(6)
1	Reedy Point.........	1.5	2.4	1.5	2.4
2	Biddle Point........	1.1	1.2	0.8	1.4
3	Biddle Point........	....	2.4	....	2.1
4	Summit..............	1.2	2.1	1.3	1.4
5	Chesapeake City....	1.9	2.4	1.6	1.7

The ordinates represent height of tide above and below mean tide level at Reedy Point, and abscissas, the corresponding velocities of current. The numbers on the abscissas indicate the hour after high water at Reedy Point. Plotted in this way, a characteristic curve is formed for each station, bringing out the relations between tidal heights and currents.

Comparison of the corresponding diagrams, Fig. 22(*a*) and Fig. 22(*b*), shows how closely the type of tide at each station is brought out by the predictions. The most striking divergence is exhibited at Reedy Point, and is due to local conditions in the canal. Beginning about Mile 1 and extending somewhat beyond Biddle Point, the canal passes through wide marshes, forming basins, with bottom elevations ranging upward from about 1 ft. below half-tide level. These are filled and emptied at each tide through the Delaware River entrance, and the resulting hydraulic currents superimposed on the normal tidal currents are the principal cause of the unusual form of the upper part of the resultant curve at Reedy Point and the somewhat similar appearance of the curve at Biddle Point.

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DISCUSSIONS

THE DON MARTIN PROJECT

Discussion

BY ROBERT S. STOCKTON, M. AM. SOC. C. E.

ROBERT S. STOCKTON,[7] M. AM. SOC. C. E. (by letter).[7a]—The construction and settlement program for the irrigation system described by the author has evidently been planned with full knowledge of the latest ideas and technical information available. The description of the work, while not detailed, indicates the thoroughness and care with which the problems have been studied.

The exclusion of certain topographically attractive areas from irrigation development on account of alkaline subsoil conditions, is most wise, and while this increases the first cost of the system the future average cost of operation and maintenance will probably be much less than it would otherwise have been. The great gain, however, will be in reducing the loss in capital investment due to lands becoming worthless from accumulation of alkaline salts. Even a drainage system as carefully planned as has been done for the Don Martin Project, will not completely protect lands with subsoil heavily impregnated with alkali, and the costs of protection or reclamation are quite likely to be prohibitive.

Stated in another way, the plans for building an irrigation system and settling the lands should include consideration of the probable condition after many years of operation and should provide, as far as possible, for the permanency of profitable production.

All those who are experienced in irrigation development are aware of the difficulty of obtaining settlers who will be able to put the lands on a productive basis before the interest and carrying charges have mounted to a point where it is no longer possible to collect them. While Governments can afford to take some loss on the ground that, in the end, an increased population and production will accrue, the plans should provide, as far as can be foreseen, for the repayment of all costs, including a low rate of interest. In most cases the development should be deferred or abandoned if it can not "pay out" on a strictly business basis.

NOTE.—The paper by Andrew Weiss, M. Am. Soc. C. E., was published in December, 1929, *Proceedings*. Discussion of the paper has appeared in *Proceedings* as follows: March, 1931, by C. H. Howell, M. Am. Soc. C. E.

[7] Supt. of Operation and Maintenance, Western Section, C. P. Ry. Irrig.-Block, Strathmore, Alberta, Canada.

[7a] Received by the Secretary, March 18, 1931.

Another point, that is most essential, is that of supervision of the farm program of the first settlers. Mr. Weiss mentions that such supervision is to be provided for all settlers on the three-year rental basis. It may probably occur that for good settlers it will be found desirable to extend the rental period to, say, five years.

The matter of proper supervision and layout of the farm management and cropping system is so important that it would be well if all land and water contracts included a definite agreement, by which the settler would be obligated to follow the directions of the farm manager or agriculturist responsible for results, for a period of five years, or until a substantial equity in the holding had been established.

It is essential of course that the agricultural supervisor shall be experienced and thoroughly qualified in every way. He should have a personality sufficiently forceful and diplomatic to obtain results with a minimum amount of friction.

Some credit for the purchase of stock and equipment is most desirable, after the character of the settler has been demonstrated as worthy, and the management has been tested as to its soundness. If funds are made available, they should be handled through a bank and by strictly business methods, as this is the point at which mistakes are most easily made, but where good management will hasten and enhance prosperity.

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DISCUSSIONS

DESIGN OF A REINFORCED CONCRETE SKEW ARCH

Discussion

By Messrs. G. D. Houtman, and J. Charles Rathbun

G. D. Houtman,[8] Assoc. M. Am. Soc. C. E. (by letter).[8a]—This paper is without doubt a valuable and timely contribution to the science of skew arch design. It is founded, among other things, on the elastic theory.

This theory is known to apply almost perfectly (up to a certain point), to a steel beam. It applies with less accuracy to a concrete beam, and with still less precision to a narrow arch rib. Presumably, it is even less applicable to a wide skew arch, involving elastic deformations within cross-sections of considerable area. Granting that a concrete section, 1 ft. wide and 3 ft. deep, bends elastically, there is no positive assurance that another section 3 by 60 ft. will bend the same way. A low building can be erected safely perhaps on a shifty foundation, but one need not trust altogether the mathematical skyscraper built on a somewhat sandy base. At the same time, the art of design has always advanced by such steps as these, and the author is entitled to commendation for his paper.

It seems to the writer that skew arches can be explained and designed with much more simplicity by a method which is frankly approximate. Let the problem be, for instance, to design the skew arch, *A B C D,* in Fig. 36. The first step would be to design, by any method, a rectangular arch of the same longitudinal span and the same width, that is, the arch, *A B′ C′ D.* This rectangular arch is next conceived to be distorted into the skew arch, *A B C D.*

The next step consists in applying the principle of least work. It is well known that Nature is economical of its efforts, and that accordingly structures act and deflect in such a way that the work done is a minimum. Obviously, the least work is done when the compression paths are of least length, and as straight as possible. There surely would be more work done in carrying a load by a roundabout or longer path. Such being the case, the skew arch, *A B C D,* tends to divide itself into a central rectangular arch, *E B F D,* and two triangular arches, *A E D* and *C F B.* There is nothing mysterious about the central rectangular arch, *E B F D.* Regarding the triangular arches, any compression path emanating from the line, *F C,* converges upon the point, *B,* which is the

Note.—The paper by Bernard L. Weiner, Jun. Am. Soc. C. E., was published in January, 1931, *Proceedings.* This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[8] Designing Engr., County Engr.'s Office, Delaware County, Media, Pa.

[8a] Received by the Secretary, January 20, 1931.

nearest point to any other point on the line, *F C*. The same reasoning applies to the other triangular arch, *A E D*, which is similar in all respects.

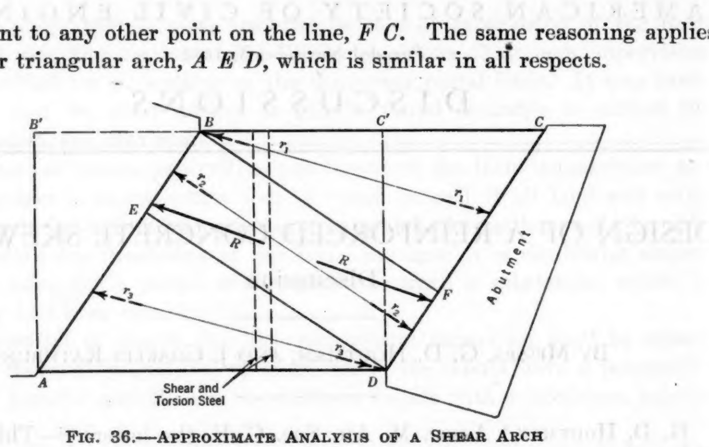

FIG. 36.—APPROXIMATE ANALYSIS OF A SHEAR ARCH

Next, bisect each of the three arches with fine lines, at the extremities of which draw the three resultants, r_1, r_2, and $r_3 = r_1$. The lengths of these three resultants are made proportional to the plan areas of their respective arches. Thus, r_1 is proportional in length or magnitude to the area, *B C F*, and r_2 is proportional to the area, *E B F D*. These three resultants are next combined to compose the final resultant, *R*. At present, this resultant is established as to direction and point of application, but not as to magnitude. It will be on the safe side to assume that its magnitude is equal to that of the total resultant acting on the center of line, *A B'*, in the original design of the rectangular arch, *A B' C' D*.

It is frankly acknowledged at this point that the procedure expounded in the preceding paragraph is not altogether correct, even if it possesses the virtue of simplicity. For instance, the magnitude of r_1 should be somewhat less than assumed herein, and, in reality, its point of application should be closer to the point, *C*, than to the point, *F*. There is also something to say regarding the different ratios of vertical to horizontal components.

The position and magnitude to the final resultant, *R*, determine the magnitude and distribution of stress along the abutment face, *D C*. Obviously, the compression stresses are greatest at *D* and least at *C*. In fact, tension may be indicated at *C*. If the compressive stresses at *D* are excessive, the arch can be thickened near that point, or, perhaps, a richer mix can be specified. The abutment is given a trapezoidal shape.

A triangular arch, such as *A E D*, obviously could not exist by itself, because the point, *D*, has no strength. It can exist only in conjunction with a neighboring arch, to which the thrust which the apex, *D*, cannot take, is transmitted by shear along the shear plane, *E D*. As far as the determination of the final resultant, *R*, is concerned, r_3 acts as shown; but it is necessary to place enough transverse steel at right angles to the longitudinal center line, to transmit the thrust, r_3, into the intermediate section.

The arch axis of each triangular arch is probably a symmetrical curve, such as a parabola or circular arc. The corresponding thrust line, however, would

not be a symmetrical curve, since there is more load on one side than on the other, of the vertical center line. Hence, the thrust line departs from the arch axis in the triangular arch. Since one such arch is the reverse of the other, certain torsional stresses are induced. Other torsional stresses are caused when the arch, for any reason, such as temperature, rib-shortening, etc., is distorted, lengthened, or shortened. The writer believes however that these torsional stresses need give no concern, provided that conservative design stresses are used and that the arch contains somewhat more than the customary amount of longitudinal and transverse steel.

Some existing textbooks state that skew arches can be designed exactly as square arches. This assertion is known to be erroneous, but it must be admitted that a large number of skew arches have been built in complete ignorance of skew principles, and that only a few have failed. Usually, failure resulted from excessive compression at the obtuse corner, and a cracking off of a section of abutment. There have been failures during construction caused by insufficient bracing of the centering. In general, the falsework of a skew arch should be braced three ways instead of the customary two. Of the existing skew arches which were designed as rectangular arches, some show peculiar cracks which indicate tension, or, at least, absence of compression, at the acute corner. Intermediate piers between skew arches are full of danger, since the skew thrusts tending to spin them about a vertical axis are doubled.

J. CHARLES RATHBUN,[9] M. AM. SOC. C. E. (by letter).[9a]—The writer has been much interested in Mr. Weiner's description of the methods of analyzing skew arches as practiced by the Westchester County Park Commission. The paper is valuable not only as a description of methods, but also because it brings the theory before the profession in a slightly different form from that used in its first presentation.[10] The numerous integral signs used in the writer's paper may have led to difficult reading for those who are more accustomed to the sigma sign for summation.

In the writer's opinion, the principal value of Mr. Weiner's paper is that it calls the attention of the profession to the fact that arches are being built successfully by this theory. When it was first published some question was raised as to the correctness of the mathematics. It was popularly believed that the stress distribution at the abutment was greatest at the obtuse corners, and when the equations failed to substantiate this idea they were not accepted by conservative members of the profession. Later, very careful and extensive experiments (unpublished) were conducted by George E. Beggs, M. Am. Soc. C. E., for the Society's Special Committee on Concrete and Reinforced Concrete Arches. The results of computations of all six abutment reactions for each of four different arches for a vertical load in a large number of positions showed substantial agreement with the results of these tests. As the report of this work for the Committee may not have been brought to the attention of all who have been interested in the subject, Mr. Weiner's paper brings before the public the fact that this method of analysis is a safe one to follow.

[9] Associate Prof. of Civ. Eng., Coll. of the City of New York, New York, N. Y.

[9a] Received by the Secretary, March 4, 1931.

[10] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 611.

In discussing the changes in practice from the original, as introduced by Mr. Weiner, the writer does not feel that these changes seriously affect any results in the final design, but are simply differences in personal practice; or, as in the use of a sigma instead of an integral sign, the difference between a theoretical formula and its practical application. Some of the suggestions, such as the use of coefficients for Equations (25), K_1, K_2, etc., and the dropping of terms of low value will occur readily to any designer.

The practice of dropping terms of low value should be treated with great caution. Those terms that are dropped in analyzing one arch, may be important in another. For example, the arches designed by Arthur G. Hayden, M. Am. Soc. C. E., for Westchester County, will permit the omission of all terms containing the moment of inertia about the u-axis and these have been dropped by Mr. Weiner. These terms are indeed very small in barrel arches, but they have a great influence in arch ribs. The writer, therefore, considers it wise for an engineer who is designing a skew arch to study the question of omitting terms as it applies to his own problem. He may find an entirely different solution than the one given. The writer also was guilty of this practice in his paper,[11] although the equations as developed are found in full in the Appendix of that paper. The same idea may be extended to some of the other "short cuts".

A designer who has familiarized himself with the original theory may find some confusion in the notation in several places in Mr. Weiner's paper. It would be less confusing to have retained the writer's original notation as nearly as possible. For example R_x, R_y, and R_z, are used as deflections due to loads by the writer while Mr. Weiner uses them as abutment reactions. The introduction of θ for the writer's $\tan^{-1}\varepsilon$, in one place, and his Δ_α, Δ_β, Δ_γ in another, might cause confusion.

The writer has found the change of signs suggested by Mr. Weiner very confusing and does not see what is to be gained by it. The sign convention introduced by the writer in his paper are self-consistent, and an engineer who follows these equations need fear no confusion. The sign of T_y was taken in the direction indicated because it is consistent with current practice in representing shear in beam analysis, and an arch approaches the beam in many cases. As it is customary to indicate positive shear in the manner shown, it is confusing to change it. If the arch tends toward the flat type the designer should not be confused by his familiarity with the beam theory. A change in the sign of the shear forces makes them inconsistent with the idea that the differential moment is shear. The change of sign of T_y and sometimes of M_y and T_z is not wise. If familiarity with the figures of a skew as shown in Fig. 2(*a*) makes it easier for an engineer to keep his work more clearly in mind, there is no mathematical objection to computing an arch as there shown, and after the work is completed sketching the results as in Fig. 2(*b*). This will avoid using a term at times as plus and at other times as minus.

Another change that has been made, which has several disadvantages, is in the order of the equations. This appears, for example, in Mr. Weiner's equations for the solution of the unsymmetrical skew (Equations (25)). He is in

[11] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 617.

error in his statement in the "Introduction" that "the theory as presented applied only to symmetrical skew frames fixed at the abutments". Much of the space in the paper devoted to the original theory, applied to the simplification of the equations due to the symmetry of the ring, but the theory as presented has been used by the writer without change for skews. If this symmetry does not exist the writer fails to see how the six equations (Equations (25)) given by Mr. Weiner, or those given previously by the writer,[12] can be simplified. Possibly the solution lies along the lines suggested by Carl B. Andrews, Assoc. M. Am. Soc. C. E.[13]

Because Equations (29) of the writer's paper[12] supply the missing terms that may prove necessary in some designs they are here stated in Equations (74) in terms of the writer's notation.

The signs, order of equations, and form of integrals have been retained as in the original paper. Note that the coefficients, K, are symmetrical relative to rows and columns. Mr. Weiner assumed the position of the load such that $z' = \epsilon x'$.

$$K_{13}\,T_x + K_{14}\,T_y + K_{15}\,T_z + K_{10}\,M_z + K_6\,M_y + K_7\,M_x = C_4 \quad\ldots\ldots(74a)$$

$$K_{14}\,T_x + K_{17}\,T_y + K_{18}\,T_z + K_{11}\,M_z + K_7\,M_y + K_2\,M_x = C_5 \quad\ldots\ldots(74b)$$

$$K_{15}\,T_x + K_{18}\,T_y + K_{19}\,T_z + K_8\,M_y + K_3\,M_x = C_6 \quad\ldots\ldots(74c)$$

$$K_{10}\,T_x + K_{11}\,T_y + K_{12}\,M_z = C_3 \quad\ldots\ldots(74d)$$

$$K_6\,T_x + K_7\,T_y + K_8\,T_z + K_9\,M_y + K_5\,M_x = C_2 \quad\ldots\ldots(74e)$$

$$K_7\,T_x + K_2\,T_y + K_3\,T_z + K_5\,M_y + K_4\,M_x = C_1 \quad\ldots\ldots(74f)$$

$$K_{13} = \int \cos^2\phi\,\frac{ds}{A} + \lambda \int \sin^2\phi\,\frac{ds}{A} + \int y^2\,\frac{ds}{I_z} + \epsilon^2 \int x^2 \cos^2\phi\,\frac{ds}{I_y} + \epsilon^2\,\lambda \int x^2 \sin^2\phi\,\frac{ds}{F}$$

$$K_{14} = (1 - \lambda) \int \sin\phi \cos\phi\,\frac{ds}{A} - \int x\,y\,\frac{ds}{I_z} + \epsilon^2 \int \left(\frac{1}{I_y} - \frac{\lambda}{F}\right) x^2 \sin\phi \cos\phi\,ds$$

$$K_{15} = -\left\{ \epsilon \int v\,x \cos\phi\,\frac{ds}{I_y} + \epsilon\,\lambda \int u\,x \sin\phi\,\frac{ds}{F} \right\}$$

$$K_{10} = \int y\,\frac{ds}{I_z}$$

$$K_6 = \epsilon \int x \cos^2\phi\,\frac{ds}{I_y} + \epsilon\,\lambda \int x \sin^2\phi\,\frac{ds}{F}$$

$$K_7 = \epsilon \int \left(\frac{1}{I_y} - \frac{\lambda}{F}\right) x \sin\phi \cos\phi\,ds = K_1$$

$$K_{17} = \int \sin^2\phi\,\frac{ds}{A} + \lambda \int \cos^2\phi\,\frac{ds}{A} + \int x^2\,\frac{ds}{I_z} + \epsilon^2 \int x^2 \sin^2\phi\,\frac{ds}{I_y} + \epsilon^2\,\lambda \int x^2 \cos^2\phi\,\frac{ds}{F}$$

[12] *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 640.
[13] *Loc. cit.*, Vol. LXXXVII (1924), p. 438.

$$K_{18} = -\left\{ \varepsilon \int v\, x \sin\phi \frac{ds}{I_y} - \varepsilon\, \lambda \int u\, x \cos\phi \frac{ds}{F} \right\}$$

$$K_{11} = -\int x \frac{ds}{I_z}$$

$$K_2 \;= \varepsilon \int x \sin^2\phi \frac{ds}{I_y} + \varepsilon\, \lambda \int x \cos^2\phi \frac{ds}{F}$$

$$K_{19} = \lambda \int \frac{ds}{A} + \int v^2 \frac{ds}{I_y} + \lambda \int u^2 \frac{ds}{F}$$

$$K_8 \;= -\left\{ \int v \cos\phi \frac{ds}{I_y} + \lambda \int u \sin\phi \frac{ds}{F} \right\}$$

$$K_3 = -\left\{ \int v \sin\phi \frac{ds}{I_y} - \lambda \int u \cos\phi \frac{ds}{F} \right\}$$

$$K_{12} = \int \frac{ds}{I_z}$$

$$K_9 = \int \cos^2\phi \frac{ds}{I_y} + \lambda \int \sin^2\phi \frac{ds}{F}$$

$$K_5 = \int \left(\frac{1}{I_y} - \frac{\lambda}{F} \right) \sin\phi \cos\phi \, ds$$

$$K_4 \;= \int \sin^2\phi \frac{ds}{I_y} + \lambda \int \cos^2\phi \frac{ds}{F}$$

$$\begin{aligned} -C_4 = \Big\{ & (1-\lambda) \int_A^P \sin\phi \cos\phi \frac{ds}{A} - \int_A^P x\, y \frac{ds}{I_z} \\ & + \varepsilon^2 \int_A^P x^2 \sin\phi \cos\phi \left(\frac{1}{I_y} - \frac{\lambda}{F} \right) d\, s \Big\}\, W \\ & + \left\{ \int_A^P y \frac{ds}{I_z} \right\} W x' \\ & - \left\{ \varepsilon \int_A^P x \sin\phi \cos\phi \left(\frac{1}{I_y} - \frac{\lambda}{F} \right) d\, s \right\} W z' \end{aligned}$$

$$\begin{aligned} -C_5 = \Big\{ & \int_A^P \sin^2\phi \frac{ds}{A} + \lambda \int_A^P \cos^2\phi \frac{ds}{A} + \int_A^P x^2 \frac{ds}{I_z} + \varepsilon^2 \int_A^P x^2 \sin^2\phi \frac{ds}{I_y} \\ & + \varepsilon^2\, \lambda \int_A^P x^2 \cos^2\phi \frac{ds}{F} \Big\} W - \left\{ \int_A^P x \frac{ds}{I_z} \right\} W x' - \Big\{ \varepsilon \int_A^P x \sin^2\phi \frac{ds}{I_y} \\ & + \varepsilon\, \lambda \int_A^P x \cos^2\phi \frac{ds}{F} \Big\} W z' \end{aligned}$$

$$\begin{aligned} -C_6 = -\Big\{ & \varepsilon \int_A^P v\, x \sin\phi \frac{ds}{I_y} - \varepsilon\, \lambda \int u\, x \cos\phi \frac{ds}{F} \Big\} W + \Big\{ \int_A^P v \sin\phi \frac{ds}{I_y} \\ & - \lambda \int_A^P u \cos\phi \frac{ds}{F} \Big\} W z' \end{aligned}$$

$$-C_3 = -\left\{ \int_A^P \times \frac{ds}{I_z} \right\} W + \left\{ \int_A^P \frac{ds}{I_z} \right\} W x'$$

$$-C_2 = \left\{ \varepsilon \int_A^P \left(\frac{1}{I_y} - \frac{\lambda}{F}\right) x \sin\phi \cos\phi \, ds \right\} W - \left\{ \int_A^P \left(\frac{1}{I_y} - \frac{\lambda}{F}\right) \sin\phi \cos\phi \, ds \right\} W z'$$

$$-C_1 = \left\{ \varepsilon \int_A^P x \sin^2\phi \frac{ds}{I_y} + \varepsilon \lambda \int_A^P x \cos^2\phi \frac{ds}{F} \right\} W - \left\{ \int_A^P \sin^2\phi \frac{ds}{I_y} + \lambda \int_A^P \cos^2\phi \frac{ds}{F} \right\} W z'$$

Equations (74) give the deflection of a skew cantilever under load and with end reactions. In the case of a skewed arch with fixed ends these deflections should be placed equal to zero and the end or abutment reactions computed.

Often the unsymmetrical skew arch has some degree of symmetry as will occur when the ring is symmetrical from the crown to the legs (in the case of an arch of the type used by Mr. Hayden), but when the legs have different lengths, or otherwise are not symmetrical. In this case the theory as originally written can be applied, except that many of the terms that will cancel in the symmetrical arch must be retained, as there stated.

In the development of the unit stresses Mr. Weiner has used a different distribution for torsional stresses. Otherwise, it is fundamentally the same as that used by the writer for several years. It might be of interest to know the difference in results between these equations, especially as Mr. Weiner states in the "Introduction" that he has compared the several theories numerically. The matter of torsional stress distribution is a very vital part of this work in many cases. Any recent research that has been conducted on this question would be very timely in the discussion of Mr. Weiner's paper by any member of the profession who has had occasion to give it special study.

Although the writer's discussion may appear to indicate that he differs from the author, in closing he wishes to state that his criticisms are all very minor and are given for the purpose of improving, a very excellent paper. Mr. Weiner is especially to be commended in proving the correctness of the method of computing the unit stresses from the forces acting across a section.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

A DISCHARGE DIAGRAM FOR UNIFORM FLOW IN OPEN CHANNELS

Discussion

BY MESSRS. LYNN PERRY, J. B. MACPHAIL, G. B. PILLSBURY, LYNN CRANDALL, AND BENJAMIN E. JONES

LYNN PERRY,[2] M. AM. SOC. C. E. (by letter).[2a]—The author is to be commended and congratulated for the work he has done in connection with this paper. It is manifest that a great deal of careful thought has been given to a problem that has confronted many engineers who have been identified with the gauging of streams.

The Chezy formula (Equation (1)), seems to be in more common use than any other, and when the coefficient is carefully chosen it is probably as accurate as any that has been proposed. However, it is based on assumptions that do not usually obtain in an unregulated stream flowing naturally. Reference is made particularly to the mean hydraulic radius, R, and the slope, S. The writer has had some experiences that have tended to destroy the usual simple faith in Kutter's value for C.

The mean hydraulic radius varies with the stages of a stream at the particular point of observation. An identical value for R may be obtained for, say, an increased depth of the water at the observation point: (a) When the increased depth is caused by back-water; and (b) when it is caused by a flood flow in the stream. It should be perfectly obvious that such conditions are not logical. It may be argued that the slope, S, would be different in a case of this kind and that this difference will cause the Chezy formula to show the proper difference in discharge. This may be answered in two ways: (1) Actual observations, limited to the writer's experience, indicate that the difference in the slope, S, does not account for the difference in the actual measured discharge; and (2) R is not a constant and should not be so treated.

This formula (Equation (1)) was proposed by Chezy in 1775 following a series of observations of stream flow. Notwithstanding the fact that it is purely empirical, hydraulic engineers have used it, almost unchanged and unquestioned, during 150 years of the most extensive engineering activity

NOTE.—The paper by Murray Blanchard, M. Am. Soc. C. E., was published in January, 1931, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

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[2a] Received by the Secretary, January 27, 1931.

since the dawn of history. Scores of engineers have lived during every generation subsequent to the time of Chezy, who are devoting more time, with more ably trained assistants, with more accurate instruments, and working on many more streams than Chezy. It certainly does seem to the writer that some engineer engaged in stream measurement and one who has at his disposal a large amount of data and some personnel, should have the initiative and courage to propose a more accurate and at the same time as economical and practical a method of measuring stream flow as Chezy.

When two observations are made in order to obtain a correct value for S at the time of reading the stage, a much more accurate value for the discharge can be computed than when the stage alone is read. The author refers to the additional expense of making the additional observation. Of course, where there is only one observation point on a stream, the expense would be doubled. Most of the large and important streams in the United States have a number of such points where the stage is read and reported every day. The writer believes that these rivers could be organized in such a way that the additional expense would be relatively small. Even if the cost is somewhat more per observation point, would it not be far better to have a smaller number of more accurate observations than a larger number of less trustworthy ones?

The author does not state whether he is making use of the well known and frequently used Kutter formula for C. When Kutter first proposed his formula for the computation of C in the Chezy formula, he did not include S; that is, his own personal observations did not indicate to him that the slope would have any effect on the constant, C. Afterward, when reports of the flow of the Mississippi River were published by Humphreys and Abbot,[3] he was prevailed upon to alter his formula to absorb certain variations that could be taken care of by admitting that the slope had some influence on the numerical value of C in individual cases. If the Kutter formula is used, the author's expression for his constant, K, in Equation (2), will not be a constant. This must be obvious because the slope,

$$\frac{F^{\frac{1}{2}}}{L^{\frac{1}{2}}} = S \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

is also to be included in C.

Assuming that the author has used some other more accurate and more reliable method for obtaining the coefficient, C, and that, therefore, his Equation (2) is correct; and if F is the total fall, in feet, in the run, L, in feet, so that Equation (6) is correct, then the value for K, as given by the author, is also correct, namely,

$$K = \frac{A\ C\ R^{\frac{1}{2}}}{L^{\frac{1}{2}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

in which, A is the area of the cross-section of the stream, in square feet; R, the mean hydraulic radius, in feet; and C complies with the requirements of the foregoing paragraph.

[3] Report upon the Physics and Hydraulics of the Mississippi River, by A. A. Humphreys and H. S. Abbot, *Professional Papers*, Corps of Topographical Engrs., U. S. Army, No. 4, 1861.

If Equation (7) is a true constant for all values of Q which the range of the observations is to cover, Equation (2) is a true parabola. Now, if this is plotted on logarithmic cross-section paper, it will be a right line; the position and direction will be fixed by the numerical values of K and the exponent of F, respectively. The numerical value of K is the point where the line intercepts the Y-axis and the exponent of F $(= + \frac{1}{2})$ is the slope of the line.

A considerable number of engineers who have had some experience in measuring the flow of water will refuse to grant that V (and, consequently, Q) varies as the square root of S. The results of much work along this line have been published.

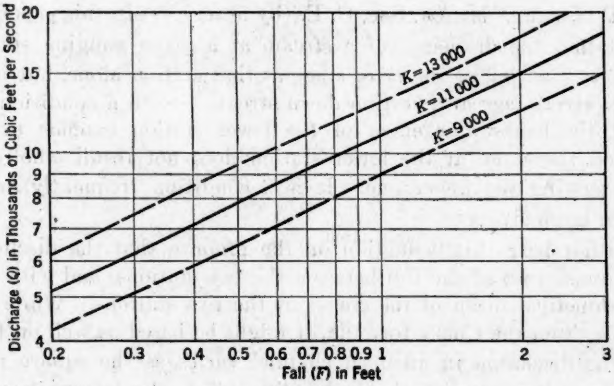

FIG. 8.

Table 2 does not give the numerical values for A, C, R or L, but values for Q and F are given for a number of cases. Taking Group A as an example, $Q = 9\,300$, $F = 0.70$; therefore, $K = 11\,000$. Taking a sheet of logarithmic cross-section paper and plotting values of F along the horizontal scale and Q vertically, the solid line on Fig. 8 can be obtained. Readings from this line will give values for Q corresponding to observed values for F as long as other factors are constant. With an increase in the stage reading, raising K, say, to 13 000, the first line above is obtained; with a decrease to 9 000, the line below, etc. A chart of this type appears to be much more direct and of much simpler construction than that proposed by the author.

If, however, Mr. Blanchard prefers to use the ratio of the square roots of the fall as abscissas, it is necessary only to consider his Equation (5) which reduces to,

$$Q_x = Q \frac{F_x^{\frac{1}{2}}}{F^{\frac{1}{2}}} \quad \text{.............................} (8)$$

Then, his base, Q, becomes the Y-intercept value, and the ratio of the square roots of the new fall to the base fall is the ordinates.

There are a great many points that might be covered in the discussion of this paper. The writer has touched only a few. He trusts that the paper will receive the attention which it deserves.

J. B. MACPHAIL,[4] ASSOC. M. AM. SOC. C. E. (by letter).[4a]—A slight simplification can be made in Mr. Blanchard's method. The coefficient, K, is a function of the physical dimensions of the reach at the stage under consideration. It varies with the mean elevation of the water surface of the reach, and a curve, therefore, can be plotted, giving the value of K for any given mean elevation.

Hence, this curve can be used with Equation (2) to plot the curves in Fig. 7. It has, however, a greater curvature than the base discharge curve proposed and, therefore, it may be less precisely definable in some cases in which experimental difficulties introduce noticeable residual errors.

G. B. PILLSBURY,[5] M. AM. SOC. C. E. (by letter).[5a]—In this paper the author points out that the discharge of a stream at a given gauging station is not necessarily a function of the river stage at that station alone, but may depend also on the river stage at a station down stream. Such a condition will occur in fact if the back-water effect of the lower station extends to the upper station, and the stage at the lower station does not result wholly from the discharge passing the upper one. It is a condition frequently arising, particularly in large rivers.

The author bases his deduction on the premise that the discharge varies with the square root of the fall between the two stations, and with a function of the arithmetical mean of the stages at the two stations. While he derives this premise from the Chezy formula, it might be based as well on the general law that the discharge in an open channel varies as the square root of the slope and as a function of the hydraulic radius, the latter, in turn, being a direct function of the river stage. If two stations are sufficiently contiguous, the slope at both will vary directly with the fall between them, and the discharge will vary as the square root of the fall. This condition will obtain, however, only when the fall between the stations is small in comparison with the river depth.

If sufficient data are available on the flow with a wide variation in fall with respect to the stage, it may be possible to derive a weighted mean of the upper and lower gauges which will afford a better measure of the equivalent hydraulic radius of the section than the arithmetical mean. In the usual case, however, the data are insufficient to justify anything but the arithmetical mean.

A diagram in the form presented by the author appears, therefore, to be based on correct hydraulic principles. The preparation of such a diagram is, however, quite a laborious procedure, and the taking off of discharge from the diagram involves estimation of proportionate intervals. A much simpler procedure is the construction of a discharge diagram for 1-ft. fall between the two stations. This diagram is used like any other discharge diagram, except that its readings are multiplied by the square root of the fall for which the discharge is desired—a very simple, slide-rule operation.

[4] With Power Eng. Co., Montreal, Que., Canada.

[4a] Received by the Secretary, January 28, 1931.

[5] Brig.-Gen., U. S. A.; Asst. Chf. of Engrs., U. S. A., Washington, D. C.

[5a] Received by the Secretary, January 31, 1931.

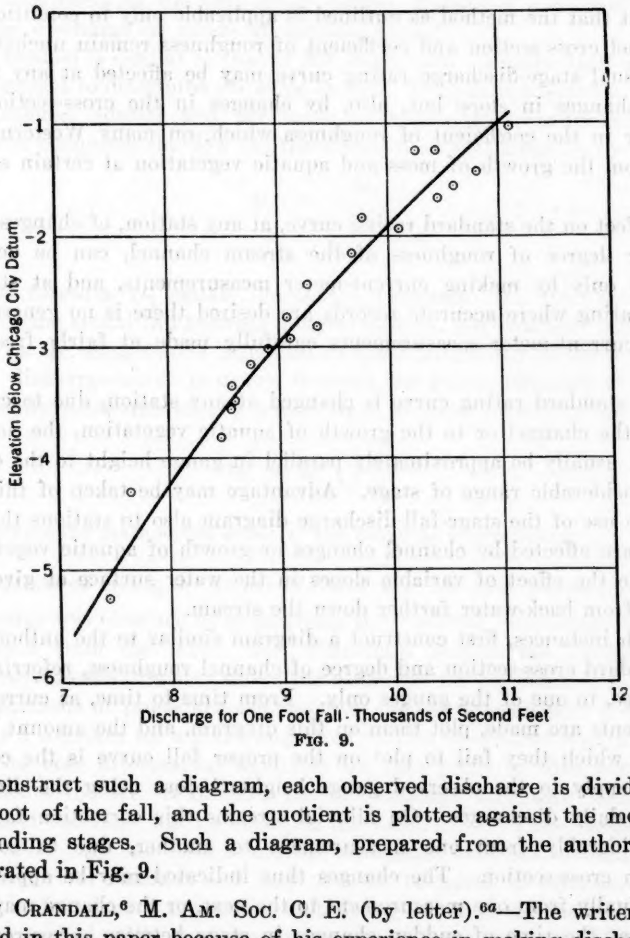

FIG. 9.

To construct such a diagram, each observed discharge is divided by the square root of the fall, and the quotient is plotted against the mean of the corresponding stages. Such a diagram, prepared from the author's data, is demonstrated in Fig. 9.

LYNN CRANDALL,[6] M. AM. SOC. C. E. (by letter).[6a]—The writer has been interested in this paper because of his experience in making discharge determinations under conditions somewhat similar to those described by the author. About 1916 he developed practically the same procedure in determining discharges on large irrigation canals in Idaho where the stage-discharge relation was affected by the operation of check-gates in the canal some distance below the measuring station. For convenience, however, the stage-fall-discharge diagrams were referred to one of the gauges only, instead of to the mean elevation between gauges as was done by the author.

The general method suggested by Mr. Blanchard, in various similar and modified forms, has been used to the writer's knowledge by many engineers in computing discharges under conditions of variable slope, and it is a valuable aid where such conditions obtain. Attention should be directed, however,

[6] Dist. Engr., U. S. Geological Survey, and Watermaster for Snake River Dist. 36, Idaho Falls, Idaho.

[6a] Received by the Secretary, February 16, 1931.

to the fact that the method as outlined is applicable only to conditions where the channel cross-section and coefficient of roughness remain unchanged.

The usual stage-discharge rating curve may be affected at any time, not only by changes in slope but, also, by changes in the cross-section of the stream, or in the coefficient of roughness which, on many Western streams, results from the growth of moss and aquatic vegetation at certain seasons of the year.

The effect on the standard rating curve, at any station, of changes in cross-section or degree of roughness of the stream channel, can be determined effectively only by making current-meter measurements, and at stations of variable rating where accurate records are desired there is no general substitute for current-meter measurements carefully made at fairly frequent intervals.

If the standard rating curve is changed at any station, due to cutting or filling of the channel or to the growth of aquatic vegetation, the new rating curve will usually be approximately parallel in gauge height to the old curve over a considerable range of stage. Advantage may be taken of this fact to extend the use of the stage-fall-discharge diagram also to stations the ratings of which are affected by channel changes or growth of aquatic vegetation, in addition to the effect of variable slopes in the water surface at given stages resulting from back-water farther down the stream.

In such instances, first construct a diagram similar to the author's Fig. 7 for a standard cross-section and degree of channel roughness, referring it, for convenience, to one of the gauges only. From time to time, as current-meter measurements are made, plot them on this diagram, and the amount in gauge height by which they fail to plot on the proper fall curve is the correction factor to apply to the observed gauge heights before using the diagram to determine daily discharges. On alluvial streams this correction factor may vary considerably from one measurement to another, due to continuing changes in cross-section. The changes thus indicated may be applied, as to time, gradually from one measurement to the next, or the change may be concentrated at the time of sudden changes in stage between measurements in the manner customarily used in the so-called "shifting-channel" method of determining discharges at stations of variable rating.

BENJAMIN E. JONES,[7] ASSOC. M. AM. SOC. C. E. (by letter).[7a]—Back-water at river measurement stations has always been a source of complication to the hydraulic engineers of the U. S. Geological Survey, as it disturbs the relation of stage to discharge. An example of this effect was encountered at the station on the Tennessee River at Chattanooga, where the stage and slope are affected by back-water from Hales Bar Dam, by the operation of the power plant, by leakage under the dam, and by the operation of the flash-boards on the dam. To meet this situation M. R. Hall, M. Am. Soc. C. E., and W. E. Hall, Assoc. M. Am. Soc. C. E., and C. H. Pierce, M. Am. Soc. C. E., at that time

[7] Chf., Power Div., Conservation Branch, U. S. Geological Survey, Washington, D. C.

[7a] Received by the Secretary, February 27, 1931.

hydraulic engineers of the Geological Survey, developed a method of determining the discharge of rivers of variable slope.[8]

Their method is similar to that of Mr. Blanchard. Messrs. Hall and Pierce used two gauges to determine the slope or fall. They then selected a fall which they considered normal and for each current-meter measurement at the station they computed the flow for the so-called normal slope by the equation,

$$Q_n = \frac{Q_1}{\sqrt{\frac{F_1}{F_n}}} \qquad (9)$$

In this formula, Q_1 equals the actual discharge; Q_n, the "normal" discharge; F_1, the actual fall between gauges; and F_n, the "normal" fall. This is the same as Mr. Blanchard's Equation (4), but in slightly different form. A "normal" discharge curve is drawn through the points obtained in this way, and the daily discharge at any given stage is then found by the equation,

$$Q_1 = Q_n \sqrt{\frac{F_1}{F_n}} \qquad (10)$$

In applying the method the letter, z, was taken as representing $\frac{F_1}{F_n}$. A table was then prepared giving the values of z and $\sqrt{z}$ for each gauge height. The true discharge was obtained by multiplying the "normal" discharge by the $\sqrt{z}$ factor taken from the table. Mr. Blanchard obtains the discharge by a series of curves each of which shows the discharge for an assumed fall.

Both methods of procedure are based on the assumption that C in Chezy's formula is a constant for a given stage and section of channel regardless of the slope. The best information available on this point is that furnished by measurements by Mr. Blanchard on the Chicago Drainage Canal, which indicate that the slope has no effect on the value of C. These measurements, however, cover only one case, where the conditions were not as complicated as in a natural river channel, and they can not be taken as conclusive. Little fault can be found with the theory underlying either of the methods and they are being applied with good results at several stations maintained by the Geological Survey.

For several years Warren R. King, Assoc. M. Am. Soc. C. E., obtained the discharge at Chattanooga by the following method: A slope-velocity curve was determined by plotting mean velocities obtained from discharge measurements at Chattanooga against the total slope between Chattanooga and Hales Bar. A stage-area curve was determined by plotting areas obtained from discharge measurements at Chattanooga against corresponding gauge heights. To ascertan the daily discharge, the velocity was first obtained by applying the mean daily slope (Chattanooga to Hales Bar) to the slope-velocity curve. The area was then obtained by applying the mean daily gauge height for the gauge at Chattanooga to the stage-area curve. The product of the area and velocity thus obtained gave the daily discharge. This method gives satisfactory results

[8] *Water Supply Paper 345-E*, U. S. Geological Survey.

except for low stages, when there is very little fall between the gauges. Theoretically, this method would seem to be applicable only where there is little change in stage.

A. W. Harrington, Assoc. M. Am. Soc. C. E., supervises several gauging stations on canals where conditions similar to back-water are caused by growing vegetation. At these stations not only the slope, but the value of C changes, and he uses the Chezy formula directly in determining the daily discharge. The coefficient, C, is computed for each current-meter measurement and plotted against a time scale. A smooth curve, drawn as nearly as possible through the plotted points, shows the variations of C throughout the season and indicates the coefficients for intervening days. The other factors in the Chezy formula are obtained from gauge-height records and the cross-section of the canal. This method is said to give good results, except when the slope factor is too small to use satisfactorily.

For conditions of rapidly changing stage, C. E. McCashin, and E. D. Burchard, Assoc. Members, Am. Soc. C. E., have used a series of curves based on gauge height and discharge, with curves for various rates of rising and falling stage. These curves are based on actual measurements during periods of changing stage, without resort to any formula.

The writer devised a method of estimating the slope due to a rising or falling stage and then obtaining the discharge for normal conditions by the use of the Chezy formula. He assumed that a rise or fall in stage travels down stream at the same speed as the surface velocity of the water. This has been found to be true of streams that have an appreciable fall and velocity. In lakes or behind dams the change in stage probably travels as a wave at a high velocity, and the effect of any change of stage on the slope is negligible. The formula as developed is, as follows:

$$\frac{Q_1}{Q_2} = \frac{\sqrt{S_1}}{\sqrt{S_1 + \dfrac{\text{Rate of change of stage}}{\text{Surface velocity}}}} \cdots\cdots\cdots\cdots (11)$$

in which, Q_1 and S_1 represent the discharge and slope at constant stage, and Q_2, the discharge at the given rate of change of stage. This method[9] has been used for many years with good results. Where large and rapid changes of stage are frequent it is better to install slope gauges and obtain the actual slope.

In using any method for computing stream flow in a section where back-water exists, it is necessary to study the local conditions that cause the back-water and to adjust the method to them. The method outlined by Mr. Blanchard is excellent and probably can be adapted to fit a variety of local conditions.

[9] *Water Supply Paper 375*, U. S. Geological Survey.

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DISCUSSIONS

LEGAL PROBLEMS INVOLVED IN ESTABLISHING SET-BACK LINES

Discussion

By Messrs. J. M. Albers, J. E. Willoughby, Horace H. Sears, George H. Herrold, Clifford C. Muhs, H. B. Cooley, Walter C. Sadler, H. J. McFarlan, Donald M. Baker, and E. G. Walker

J. M. Albers,[4] Esq.—It appears from a study of the Court decisions affecting zoning, and particularly from some of its special phases, that the legality of regulations creating building set-back lines is established. This statement can be accepted without undertaking a lengthy discussion of it, because supporting decisions may be found all the way from the United States Supreme Court down to the Lower Courts.

In the Chicago, Ill., Region, city engineers and planners are especially interested in the legality of set-back line legislation from the standpoint of its possible application to street widening. They are well aware of the contention that such an application of this principle encroaches upon the realm of eminent domain in that it may constitute a taking of private property for public use without compensation. There seem to be plenty of advocates on both sides of the case of police power *versus* eminent domain when street widening is being discussed. Is not an owner deprived of the use of his property by a zoning set-back line just as much as he is by a set-back line for future widening? Is not the establishment of a building set-back line on a future wide street just as much in the interest of public health, safety, convenience, and welfare as any other set-back line? The preservation of these things is the objective of police power. The actual taking of land can be left until the physical work of widening is to be undertaken. In the last few years the application of the police power has found a much greater field of usefulness in zoning legislation, and the large number of zoned communities in the United States bear witness to the success and propriety of this form of regulation. Practically all the existing zoning ordinances contain set-back provisions, and these are established in the interest of public health, convenience, and welfare.

Note.—The paper by Clifton Williams, Esq., was presented at the meeting of the City Planning Division, Milwaukee, Wis., July 11, 1929, and published in January, 1931, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

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Streets, adequate as to location, width, point of origin, and destination are of tremendous importance to proper civic development. Public safety and welfare are perhaps, to-day, more dependent upon the proper provision of streets than upon any other single function of government. The rapid growth of urban communities is continually demanding streets wider than those originally laid out. The larger the community the more difficult it is to secure the wider rights of way. The cost of condemnation is in many cases a greater burden than the community is able to bear and, therefore, these highly necessary improvements are delayed and sometimes abandoned to the detriment of the entire community.

At the time when he published the plan for the City of Chicago, the late Daniel H. Burnham recognized the advantage of set-back lines for future street widening and recommended that the city acquire sufficient front-yard space to prevent future encroachment. If the necessity for wider streets is admitted and, at the same time, if the fact that the cost of condemnation will prevent much necessary improvement also is admitted, then city planners must seek further for a more economic method of accomplishing this work. The establishment of set-back lines now, under the police power, is the only practical method by which the widening of many traffic arteries can be secured in the future when greater width will assuredly be required. It is painless, economical, and has the decided advantage of making necessary a comprehensive plan for future street improvements and civic development.

The speaker believes that sufficient foundation has been laid to warrant some procedure along this line. The United States Supreme Court has ruled (Gorieb *vs.* Fox) that,

"It is hard to see any controlling difference between regulations which require the lot owner to leave open spaces at the sides and rear of his house and a regulation which requires him to set his building back a reasonable distance from the street".

The Cleveland, Ohio, set-back ordinance has been sustained as a proper exercise of the police power, as was that of the Town of Windsor, Conn. In the last-mentioned case the Court decided that such legislation was not in violation of constitutional rights. The Courts of Wisconsin, California, Ohio, and several other States have all sustained set-back regulations. In the case of Kaufman *vs.* City of Akron, Ohio, the Court declared that the burden is upon the plaintiff; it is for him to show that set-back lines bear no substantial relation to public health, convenience, and welfare, before they can be declared invalid. In another Ohio case (Pritz *vs.* Messer) the Court ruled that the validity of the set-back law must be viewed in the light of probable future development. The speaker does not believe that any of the outstanding set-back decisions bear specifically on the use of set-back lines for street widening, but an array of opinion is being built up which, it is hoped, will finally sustain it. The construction of costly buildings in the bed of new streets must be prevented if there is to be any hope of progress toward adequate street systems. It appears that the regulation of the use of property to protect a major street plan is acceptable as to purpose, but there is a difference of

opinion as to whether or not this should be accomplished through the police power or the right of eminent domain.

An examination of the Court decisions to date indicates that set-back laws are rapidly being sustained by the high Courts of the land as a proper exercise of the police power, and that resort does not have to be made to the power of eminent domain with its attendant prohibitive expense. If such legislation is worked out by the governing body on the basis of a well-defined policy, after a careful study of the entire area affected, and after due notice and hearing so as to avoid all charges of discrimination, arbitrariness, and unreasonableness, the Courts will not interfere with such legislation. Such enactments will speedily justify their own existence, and there is a growing sentiment in favor of them in the Courts as well as in public opinion. When such legislation is predicated upon public health, safety, convenience, and welfare, it is brought within the scope of a proper exercise of police power.

In the Chicago Region many communities are now protecting future street widenings by the present establishment of set-back lines for this purpose. Some of these communities have passed ordinances creating such lines on specific streets, others are obtaining wider streets through agreement between the property owners and the city and village government. Still others have included the major street system with its required set-backs as a part of the zone ordinance. This latter seems to be the best method in that the city then has the opportunity to use the machinery set up in the Board of Zoning Appeals and this body is in a position to adjust those difficulties which are bound to arise in so comprehensive a piece of legislation.

J. E. WILLOUGHBY,[5] M. AM. SOC. C. E. (by letter).[5a]—One purpose of the written Federal Constitution is to limit the powers to be exercised over the people by Federal authority. That particular sovereignty advocated by the author is denied by the Federal Constitution, but (in the original draft) not to the extent which the debate in the various States (particularly Virginia) on its adoption developed was necessary for the protection of the people. Hence ten amendments were introduced immediately after the Constitution had become effective and, in due course, they were adopted. Of these, the Fifth Amendment had particular reference to protection of private property against seizure for public purposes. This protection was further strengthened by the later adoption of the Fourteenth Amendment.

What the author is suggesting is a change in the Federal Constitution to be procured through such propaganda as will induce the Courts to issue decisions to the effect that private property, if taken under the guise of the police power, is not a taking such as is prohibited by the Fifth and Fourteenth Amendments. It may be assumed that the author is of the opinion that his purpose cannot be attained by repeal of the Fifth and Fourteenth Amendments although such a repeal probably would authorize public authority to take private property for public uses without compensation and without due process of law. The writer is not in sympathy with the suggestion. The exercise of the police power is properly used to further the comfort and safety

[5] Chf. Engr., A. C. L. R. R., Wilmington, N. C.

[5a] Received by the Secretary, January 22, 1931.

of the people. That comfort and safety can be obtained without confiscation of private property.

The arguments advanced by the author as warranting a Court decision for the extension of the police power to include the taking of private property without compensation are not sound in that proper distinction is not made as between the principle which permits the lawful limitation of the height of a building and denies as unlawful the taking of an area of land occupied by the building. Every land owner of the fee of a part of the surface of the earth has the right to use it for lawful purposes, provided that such use does not take away in any particular the property rights of an adjoining owner. Among such property rights are sunlight and air. Whenever the height of a building destroys the right of another to full enjoyment of sunlight and air beyond the area covered by the building, then the height of that building can be fixed by law so as to remove the invasion of such a full enjoyment. The regulation of height of a building is not the taking of the property of the owner thereof, but is a prevention of the owner of the building from taking the property of another; which "another" may be the user of the public streets. The quarantine of a man with a contagious disease is not the taking of that man's property in form of wages, but is the prevention of that man from taking away the right of his neighbor to have freedom from infection.

The municipal authorities under the police power can properly decree a set-back line for building construction, and can forbid the use of the area between the set-back line and the edge of the street under the lawful principle of preventation of the lot owner from using his property to the detriment of others. So long as the municipality refrains from using as municipal property any part of the area between the street line and the set-back line, no property is taken; the rights of the adjoining others are protected, and nothing more. The actual conversion of such area to street purposes is the taking of private property for public uses which the Federal Constitution now forbids. The rights which develop from long-continued use are conditioned on another principle of law—open and notorious possession and use presume title in the holder and the user. Such is not a taking under the police power.

The fact that property needed for street widening is valuable and may cost the public much money, can never be a warrant for seizure without compensation. Possibly such was the thought and practice of Attila.

HORACE H. SEARS,[6] M. AM. SOC. C. E. (by letter).[6a]—The inclusion of "set-back lines" in future street-widening zone laws is worthy of consideration for cities where this future need is anticipated. The police power exercised as an incident to present zoning legislation has been limited by Court decisions to future, as distinct from past, activities by the owners of vested property rights both real and personal.

State and Federal Constitutions assure the owner of real property rights that no retroaction laws shall be passed. The claim concerning the effect of destroying vested corner-lot rights in the larger cities by legislating arbitrary

[6] Cons. Engr. and Attorney-at-Law, Hastings-upon-Hudson, N. Y.

[6a] Received by the Secretary, February 5, 1931.

destruction of improved property values through the exercise of police powers incident to new zoning laws, as proposed in this paper, is fallacious. The attempted deduction of the legality of thus interfering with vested rights of property owners as being in line with fire and flood prevention, involves the same fallacy as attempting to prove that a dog is a horse for the reason that a dog is an animal and a horse is an animal, hence, they are equal.

It is here submitted that the selection of the decided case of Hayes *et al v.* Hoffman (192 Wis. 63), and the same case (211 N. W. 271, decided in 1927), is not "on-all-fours" with the argument stated by the author for the following reasons stated in a syllabus of that opinion; that is, the Lower Court, upon trial, found that the proposed apartment building owned by the defendant would violate the provisions of the zoning ordinance in overstepping established lines on all four sides of the property. Furthermore, the area to be occupied by the apartment house as well as the area occupied by the defendant's frame house at the rear of the premises, was greater than the maximum permissible area allowed under the terms of the ordinance.

In its finding, the Court states:

"It is our own conclusion that the ordinance is, in the respects here considered, a reasonable, valid, and constitutional enactment. It is appreciated that there are other provisions of the ordinance the validity of which may be the subject of future challenge. It is to be understood that no opinion is expressed with reference to any features of the ordinance except such as are herein treated."

The Appellate Court further states (192 Wis. 63) concerning the failure of defendant to plead his constitutional rights:

"It is the duty of a person who claims that an ordinance invades rights guaranteed by the Constitution to raise that question at his earliest opportunity. Taking part in a proceeding which fixes his liability under the ordinance without raising any question as to constitutionality of the ordinance is a waiver of the right to raise that question subsequently.

"The plaintiffs were given an opportunity to meet this issue (as to constitutionality) in the Court below. The record was made up without reference to that claim made here for the first time. We cannot entertain it."

No attempt will be here made to discuss the author's suggestion under the heading, "The Engineer in Relation to Legislation Concerning Set-Back Lines", or that there is need for engineers to inform Courts as to the enormous cost of putting city streets which are congested by motor traffic in safe condition; nor for engineers to inform the Courts that street relief sought in "set-back lines" is "as much a problem of sovereignty as fire-fighting, quarantine, flood relief, nuisances, etc."

The exercise of the right of eminent domain and the use of recent legislation in street widening through the medium of excess condemnation gives far more assurance of stability to property values and hence to collection of taxes; and if these recognized remedies for the exercise of the sovereign power of the State are properly conducted, the financial relief will be had which Mr. Williams indicates is much needed.

Zoning laws directed to future possible taking of realty for street widening, leave values inert, and the improvement of city street frontage in a stagnant condition. The decided cases are numerous which uphold the contracts that a

person makes with the State, or its agencies, such as a municipal corporation. Built-up blocks with narrow streets which require widening, come within the classification of vested rights, the disturbance of which by reason of new legislation under the guise of "exercise of the police power" invites costly and prolonged litigation.

The exercise of the police power and the interference with vested rights of realty owners in connection with zoning laws and city ordinances is discussed by the Court at great length in the recently decided suit of Jones *v.* City of Los Angeles (Cal. Supreme Court, December 31, 1930).[7] The opinion is replete with citations of decided cases and the rules of law controlling the use of the zoning laws, police powers, and the retroactive effect incident thereto. Said the Court, in part, in its concluding statements:

"No general test can be derived from these or other cases, all of which reiterate the well-worn doctrine that each one must be decided on its particular facts. But we think that there is a decided difference between the ordinance which operates to limit the future use of property, no matter how great the impairment of its value, and one which requires the discontinuance of an existing use. * * *

"If the city desires to abolish the non-conforming use, this may be a legitimate object of the police power, but the means of its exercise must not include the destruction of the property interest with compensation. The words of Mr. Justice Holmes in Penn Coal Co. *v.* Mahon, 260 U. S. 393, 413, are very much in point; 'As applied to this case the statute is admitted to destroy previously existing rights in property and contract. The question is whether the police power can be stretched so far.'

"Even where the destruction of private property is warranted by the vital public necessity, it is sometimes the legislative practice to compensate the injured owner. Such was the legislative intent in the eradication of bovine tuberculosis by the destruction of diseased cattle (see Patrick *v.* Riley, 79 Cal. Dec. 379).

"Our conclusion is that where, as here, a retroactive ordinance causes substantial injury and the prohibited business is not a nuisance, the ordinance is to that extent an unreasonable and unjustifiable exercise of the police power."

GEORGE H. HERROLD,[8] M. AM. SOC. C. E. (by letter).[8a]—Establishing set-back lines to create front yards is a common procedure under zoning. They are not difficult to establish because the people like set-backs in residential districts. It is done under the legal fiction of providing light and air and of promoting health and general welfare. Taking a bath is theoretically a matter of personal hygiene, but practically a bath is taken because it makes one feel good; a person likes it. Home owners like set-backs.

Set-back lines for the purpose of widening streets come under a different legal fiction. It is said to be a taking of land for public use without compensation. Mr. Williams' contention is that both are within the field of Government sovereignty; that if set-back lines can be established to create front yards for light, air, health, etc., they can be established for the purpose of widening streets because of congestion; that if limit can be established for the height to which a building can be built by zoning, the ground area used can be limited also. Following this same line of thought, the writer

[7] Reported in the *U. S. Daily*, February 2, 1931, p. 8.
[8] Managing Director and Engr., The City Planning Board, St. Paul, Minn.
[8a] Received by the Secretary, February 9, 1931.

would suggest that, as the speed of vehicles is now limited in congested districts in the interest of safety, it might also be possible to determine and limit the number of vehicles and people that would be permitted to use a given street in the interest of safety.

The street-widening problems of the cities of the United States are staggering. Costs of street widening range from \$1 000 000 to \$9 000 000 per mile, omitting such projects as Michigan Boulevard and the Wacker Drive, in Chicago, Ill., and the Fairmont Parkway, in Philadelphia, Pa., which cost far more than this. These costs are due to methods of procedure in condemnations. These methods make possible the building up of excessive values for the property to be taken. The values are so great that they absorb the assessments for benefits, legal fees, and other items, and make the citizens at large (through bond issues) pay a profit to the owner. Later, the owner also receives the benefit of the increases in value that always follow a street widening. This method cannot go on.

The City of St. Paul, Minn., has a retail street, 60 ft. wide between property lines, 40 ft. of pavement between curbs, 10-ft. walks, and a double-track car line occupying 19 ft. of the 40-ft. roadway. Real estate values on this street are the highest in the city. Congestion is intolerable and has been so for years; no more people or vehicles use the street to-day than ten years ago; they cannot because there is no room. Three department stores on this street have aided their own situation by arcading, each placing a 16-ft. walk in their building parallel to the street sidewalk. The Planning Board made a study of the cost of widening this street. It found that if by agreement the property owners would widen the street to 86 ft., each paying his own costs, cutting back his building, putting in a new front, remodeling the interior, building a new 15-ft. sidewalk and paying for widening the pavement 8 ft. in front of his property to create a 56-ft. roadway, the actual cash outlay in round figures would be \$1 000 000. The owner would be reimbursed in a few years by the increased value of his property and increased rentals due to his property having greater accessibility on the widened street.

If this street was widened by the City under the power of eminent domain, assessing benefits and damages, the cost would amount to \$6 500 000. This is the cost of the present procedure under methods which are common practice where land and buildings are acquired under condemnation proceedings.

Referring again to Mr. Williams' interesting discussion on the use of sovereignty: It takes a long time to change an established legal precedent. This, therefore, seems to the writer to be the proper time to call attention to a proposal made by C. E. Grunsky, Past-President, Am. Soc. C. E., before the City Planning Division of the Society a few years ago. He suggested establishing a building line for widening the street, condemning the strip of land lying between this building line and the street line separate and distinct from the buildings, paying the owners for this land, but compelling each owner to pay the city 6% interest on the amount paid him as long as his building stood on the land. When the owner sets his building back to the new line, he would stop paying interest. In the meantime, the city would have been paying, say, 4½% on the bonds which it floated to pay the owners for the land.

The City of St. Paul has a provision in its charter for carrying out the first part of the procedure, that is, the condemnation of land separate and distinct from the buildings thereon. The objection to using this method is that the owner of the building then has the use of the land for which the city has paid until such time as the Council may take physical possession, and if the City takes possession before the owner has seen fit to set his building back, it must then pay for the building. Mr. Grunsky has thought this idea through to a more logical conclusion, and his proposal is worthy of more consideration than has yet been given it.

CLIFFORD C. MUHS,[9] M. AM. SOC. C. E. (by letter).[9a]—The usual legal bias seems to be that the earth was created to demonstrate the beauties of the Illinois statutes. The viewpoint expressed in Mr. Williams' paper is so different from this that it is decidedly refreshing.

Obviously, the more industrialized (which must also apparently mean the more thickly settled and migratory) a people become, the more surveillance is required to prevent irresponsible parties from taking advantage of their comparative obscurity. Among nomadic peoples anarchy can, and often does, exist as an ideal system. Among pastoral peoples, some faint beginnings of rules of conduct are discernible although there may be nothing more ponderable than public opinion to enforce them.

As civilization progresses, the code of conduct must increase in scope and complexity with a corresponding increase in economic life. It must be extended so as to protect the average person from the ruthless, irresponsible, or obdurate individual whom public opinion cannot now effectively curb.

Legislators, who have more frequently been chosen for their dynamic personalities (real or assumed), have been quick to recognize this trend. If their ability and familiarity with the ordinary economics of life had equalled their zeal, there might now have existed a fairly workable code in spite of the ability of organized minorities to advance their own interest in law at the expense of that of the public.

In the judicial branch of Government the case is different because the candidates are selected for entirely opposite characteristics to such an extent that they might be designated the "high priests of the objectors". To the utter negation of all progress, these objectors seem to abide by the slogan, "Whatever is is right. What has not been done cannot be done." Such men would say that justice is immutable and what was just in Lycurgus' day was just now, ignoring what justice might be essentially.

With such an attitude, legal procedure must always lag a century behind that of its contemporaries, thus becoming indeed the check upon Government it is often proclaimed. Such an apposite as that taken by Mr. Williams is, therefore, the more worthy of lay support, and of such support undoubtedly the most effective should be that of the engineer. The engineer is as little concerned with the past as the lawyer is preoccupied with it, and his hypothesis is that "anything can be accomplished."

[9] (Muhs & Co.), Chicago, Ill.

[9a] Received by the Secretary, March 9, 1931.

In special assessment work the writer has had ample occasion to view the chronic objector in action, even when his interests are being advanced by the project to which he objects and when the compensation in money and enhancement far outweighs the loss.

If then the Bench can be induced to view, with less sympathetic eye, those who see in obstructionist tactics merely another chance to advance their own interests at the expense of others, it will be a consummation devoutly to be wished.

H. B. Cooley,[10] Assoc. M. Am. Soc. C. E. (by letter).[10a]—The tendency of the Courts to treat the establishment of set-back lines as a function of the police power of the municipality may be traced through a series of decisions of the Supreme Court of the State of Louisiana.

In 1915 (Clavo *v.* City of New Orleans, 136 La. 480, 67 So. 338), and, again, in 1917 (State *ex rel* Blaise *v.* City of New Orleans, 142 La. 73, 76 So. 244), the Supreme Court of Louisiana held that the Charter of the City of New Orleans did not give the Municipal Council the authority to enact a so-called zoning ordinance. The sections relied upon were those which granted the usual police power to a municipality. The following year the Legislature passed an Act enabling cities of more than 50 000 inhabitants to make such laws to regulate the kind, style, and manner of constructing buildings on certain streets and to permit or prohibit the establishment of business within designated limits. The scope of this Act was enlarged and its provisions were extended to all municipalities, regardless of size, by the Constitution of 1921.

Shortly afterward, a case reached the Supreme Court, which originally had been brought under the ordinance declared unconstitutional in the Clavo Case, but which also came under a later ordinance passed after the Act of 1918. On this occasion, the Court held that,

"If the municipal council had not enacted Ordinance No. 6689 but had relied upon Ordinance No. 5599, we might, in respect for the expressions of the legislature and of the Constitutional Convention, and in the light of recent decisions of the Supreme Court of the United States and of several State courts of last resort, depart from the rulings made in Clavo's Case and repeated in Blaise's Case, and maintain that Ordinance No. 5599 was within the authority granted in the City's Charter and was a valid exercise of the police power".

While it was not necessary for the Court to over-rule its previous decisions, at the same time, it shows clearly a much broader view on the subject of the police power as related to zoning laws. (State *ex rel* Civillo *v.* City of New Orleans, 154 La. 271, 98 So. 440.)

In 1928, the question of a set-back ordinance was raised. This ordinance prohibited the City Engineer from issuing a permit for the erection of a building within 15 ft. of the property line of St. Claude Avenue, one of the principal thoroughfares of New Orleans, between certain streets. The Lower Court declared the ordinance unconstitutional, because the power to enact such a regulation was part of the power of eminent domain, and, therefore,

[10] Napoleonville, La.

[10a] Received by the Secretary, March 9, 1931.

could not be exercised without payment of adequate compensation to the owner for the taking of his property for public use or benefit. The Supreme Court in over-ruling the decision of the Lower Court held that a set-back ordinance does not constitute a taking of private property for public purpose under the power of eminent domain, but, like the so-called zoning ordinances, is merely an exercise of the ordinary police power. (Sampere *v.* City of New Orleans, 166 La. 776,117 So. 827.)

In the last case the Court cited Gorieb *v.* Fox (274 U. S. 603). A city ordinance of Roanoke, Va., created set-back lines to be at least as far from the street as the line occupied by 60% of the existing houses in the block. In this case the term, "block", denotes that portion on the same side of the street as the proposed building and bounded by nearest intersecting streets to the right and left of the building site. The Supreme Court of the United States held that there was no difference between ordinances prescribing the height of buildings to be erected and the area to be left open for light and air and in aid of fire protection, which have been held valid under the Federal Constitution, and ordinances which required an owner to set back his house a reasonable distance from the street. (In this case the set-back was equal to 34 ft.).

In two recent zoning cases (State *ex rel* Dema Realty Co. *v.* McDonald *et al* 168 La. 172, 121 So. 613; and State *ex rel* Dema Realty Co. *v.* Jacoby 168 La. 752, 123 So. 314), the Court upheld an ordinance which required business to liquidate and move out of the zoned area within one year, as it did not deny defendants of property without due process of law.

In view of the tendency shown by the cases cited, it does not seem a very "far cry" to the time when, as Mr. Williams suggests, such set-backs may be created. Owners of buildings erected over the set-back line will then be given a limited time to move back the proper distance and after that period the property will be used by the municipality under its "police power", for the purpose of handling the ever-increasing and, consequently, more dangerous traffic problem.

WALTER C. SADLER,[11] M. AM. SOC. C. E. (by letter).[11a]—This paper is an exceedingly interesting presentation of a proposal to extend the police power of the State to the acquisition of property for the widening of congested thoroughfares. The present discussion will be confined to two aspects of this problem: First, the legal issues involved; and, second, the desirability of such a doctrine.

It is quite clear that the police power has never been used in this type of condemnation of land, or property rights, because all cases of such power bear some element of emergency, where time of action is of the essence. In the case of removal of oyster-bed stream obstruction to stop an epidemic, it is imperative that the obstruction be removed at once without recourse to the slower alternative of applying a suitable chemical to the water. In the case of fire, it is necessary to destroy the intermediate block at once, to prevent the

[11] Associate Prof., Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

[11a] Received by the Secretary, March 18, 1931.

spread of the conflagration; and this is also true in the case of the destruction of gambling devices, to prevent the spread of this vice. Each overt act is taken to prevent a greater harm to the public, with immediate action essential. The nature of the remedy suggested by Mr. Williams bears no such time element of emergency. In fact, the suggested program involves from five to twenty years for securing the proposed result. Surely that period of time is sufficient in which to find some other engineering solution of the problem, such as the construction of an elevated highway, subway, or detour. In other words, the police power of the State is an emergency device with no proper application to a long-term project.

The only justification for the exercise of this power lies in the preservation of public life. Surely it could not be applied to the case of "public convenience", or the doctrine could then be extended to grade-crossing eliminations, etc. If the doctrine is to be applied to the preservation of life, wherein lies the emergency of a remedy twenty years in the future?

The case of levee work in the South appears to bear out the doctrine suggested by Mr. Williams, but even this group of cases does not seem applicable, for the land there is held under the Napoleonic Code. In that territory, a man holds the property with the understanding that his rights in land are limited in this respect. In the present case of set-back lines, the abutting owners on the congested thoroughfare purchased their property secure in the knowledge that their rights would not be taken without compensation except in emergency cases. The situations are easily distinguishable.

The "zoning case" cited presents a particularly interesting situation. It is a clear case of a taking of property. In fact, it is just as much a taking as though the State had established an easement for highway purposes, the owner retaining the fee. However, the results obtained are due to the peculiar shape of the ground, and its location, and the precedent does not seem to be a very strong one upon which to justify the taking of extensive abutting property rights. Certainly the zoning of 15 ft. on the lawn of a lot in the center of a block would not be suggested as justification for the taking without compensation of that 15 ft. for road purposes.

It is the writer's understanding that Mr. Williams is not trying to establish his doctrine as existing law, but merely as a near step to existing law. The present discussion is offered merely to question the proximity of his doctrine to existing law. It is felt that the taking of property without compensation is quite a long stride.

It would seem appropriate to question the very desirability of the taking of land without compensation. It is certainly shocking to the American concept of property rights. The inviolable nature of property is accepted throughout the land, and the average American understands the essence of the protection. In fact, it is so thoroughly embedded in the American concept of government that should the Courts adopt Mr. Williams' theory and interpret the law as authorizing the taking of this property without compensation, every law-making body in the United States would come to the aid of the abutting land owner.

Mr. Williams has suggested that the problem is an extensive one, involving enormous sums of money, and offers this as a partial justification for the plan. For every square foot of land acquired there must have been an equal amount lost by some individual. The expense to the community is scarcely a justification. In one place it was stated that the cost must be borne by the municipality; that, in the final analysis, the problem is not one of State or Federal concern. The Courts have stated that grade-separation problems are distinctly of State concern. The law itself takes cognizance of this situation. For instance, statutes in New York provide that the State pay 49% of the expenditure and the county 1% in the grade-separation project within a city. Other States contribute in varying amounts. The same relationship should apply to the widening of existing streets used as State trunk lines. It is certainly logical in view of the amount of automobile weight taxes and gasoline taxes, which go directly into the State Treasury. It would appear that the Engineering Profession might do better to sponsor a re-apportionment of the expenditure of these enormous State taxes, than to endeavor to secure legal sanction to the establishment of the doctrine of "taking of property without compensation".

Mr. Williams should certainly be commended for his able and interesting presentation of a proposed solution to a serious transportation problem.

H. J. McFarlan,[12] Assoc. M. Am. Soc. C. E. (by letter).[12a]—In this paper Mr. Williams is making an appeal for engineers to lend their influence in convincing the Courts to accept his "new idea, based on the age-old functions of sovereignty". If engineers have and are to maintain any influence it is only because they may have some ability in making an adequate analysis and a reasonable solution of the problems that come to them. It is the purpose of this discussion to consider further the function of sovereignty, to refer in a general way to some of the cases cited by Mr. Williams, to suggest a different point of view, and to submit an argument in its defense.

There can be little question about the right or power of sovereignty to do whatever is necessary for the public good; but there can be many questions as to what constitutes the public good. That the function of sovereignty extends back beyond the time of any formulated expression, is true. In fact, it might be called a "natural" function, first exercised unconsciously when men began to live together, and first exhibited when the power of the group was exercised coercively.

That the widening of streets is a matter of serious public concern is not denied. This admission is also a concession that this problem is a function of sovereignty. More than that, it is agreed that it is a function of sovereignty (and a necessary one) to solve all serious public problems, and that these are rapidly increasing in modern society. Little more than a century ago highway construction and maintenance were not recognized as of public importance. To-day, sovereignty is exercised in varying manner and degree ranging from actual ownership and control of highways to regulation and quasi-regulation of numerous other public utilities.

[12] Asst. Prof. of Surveying, Univ. of Michigan, Ann Arbor, Mich.

[12a] Received by the Secretary, March 19, 1931.

A change in the nature of vital functions can be foreseen in the future. Many now considered as private, will be recognized as public. If industry is speeded to the point of producing nervous wrecks, sovereignty will intervene. The beginning phase of this is already appearing in safety laws, the regulations of working hours for women, the establishment of food laws, and numerous other types of regulatory measures. If existing economic devices do not or cannot be made to assure the means of livelihood to the masses, sovereignty will re-adjust the devices. These are implications that must be drawn if one is to consider seriously the functions of sovereignty. These are things that the engineer should be concerned about because, unless there is some careful and precise thought exerted, the adjustments are likely to be made in a bungling way. Sovereignty is a "natural" power and Nature has little concern for economy, as men conceive it. If some measure of human economy is to be secured it is by directing sovereignty in such a way that the predictable effect will be desirable. It is the business of engineers working in the physical field to direct forces to the accomplishment of desirable ends. The attempt to interest engineers in getting an understanding of the problems of sovereignty is laudable.

The legal attitude is to seek authority in the shape of precedent to justify a conclusion. Cases in which sovereignty confiscates rather than condemns are cited in support of confiscation for street-widening purposes. It would seem that the analogies between cases cited and the proposed confiscation are not entirely sound. The Fourteenth Amendment to the Constitution of the United States is a formulation of a principle that was recognized before constitutions, as being necessary to the continuance of law. The demand on the part of laws for socially acceptable conduct is not in contravention of that principle. Neither can it be expected that by law one can be protected against the vicissitudes of Nature. It is true that as society has become more complex there has been a growing tendency toward restricting the use of private property. If an increase in social complexity makes it necessary seriously to restrict, or to confiscate, as a common thing, sovereignty may re-adjust values to the extent that all rights in property may become social.

The taking of property for street widening would seem a step in this direction. It does not seem analogous to the prevention of fire, flood, or disease, and even seems somewhat different from zoning. The precedents cited are of two classes. One is of the emergency type due to a condition caused by what is known in law as "an act of God." The other is a demand for socially acceptable conduct. There is great necessity when a building needs to be destroyed to save the spreading of fire. The right of a quarantined man to expose the community is frowned upon socially. The right to build a factory in a residence district is also denied. As will be argued subsequently, social insurance for the unfortunate ones suffering loss might not be unreasonable. If a set-back line is established before property is developed, this is of the nature of zoning. This, in effect, is done now through the power exercised by planning boards. No one arguing for zoning would be so bold as to suggest the destruction of factories to provide a residence district. The demand for socially acceptable conduct is vastly different from upsetting values that have

accrued through legitimate use. The proposed confiscation is not necessary because of an accident of Nature, it is not a demand for the regulation of conduct (in an active sense), and it is fraught with dangers because the "when" and "where" of its being practiced is within the control of men. This latter fact also makes it different under the "equal rights" law from all the cases cited, except perhaps zoning, and that is not retro-active, so to speak, against improvement.

The business of the engineer is to conceive the desirable and then to set about to find ways and means of accomplishment. He does not ignore prejudice and precedent, because an account of these must be taken in making his evaluations. Neither does he accept what has been done as authority for new endeavors. To him the worth of an idea is dependent upon the effect it will have. This is what is known in the terminology of modern writers on the philosophy of science as the operational point of view. This is the point of view that has speeded the production of the marvelous creations of modern science. It is responsible for deliberate invention in contrast to accidental discovery.

It is true that an engineer works in the so-called field of the exact sciences. It does not follow, however, that the operational point of view cannot be held when dealing with problems that have to do with human reactions. Indeed, the difference between the two fields of endeavor is due probably only to the difference in precision with which the data are obtainable. One frequently reads or hears an appeal made for the engineer to lend his aid to the solution of some grave problem. If he is to have an effectiveness comparable with what he has had in the solution of his own problems, it will be because he carries over his scientific type of thinking to this other field. It is not conceivable that he would be likely to be more successful with lawyers' law, sociologists' sociology, or economists' economy than men who have made these studies their life work. It is within the realm of imagination, however, to think that the engineer—because he is active in a field that demands that the worth of an idea be tested by the effect it has—may be able to lend valuable aid to those engaged in the solution of social problems. Unless he is cognizant of the philosophy of science, he is likely to lend his support upon recommendation of the specialists in the other fields, and thus bring no more influence to the solution of the various problems than any other citizen.

The burden of the argument in favor of confiscation has been carried by a citation of precedent, aimed at persuading that the solution is in a realization of the fact that it is a problem of sovereignty. It seems like a begging of the question to assume that confiscation is the only remedy that sovereignty suggests.

The engineer, when called upon to attempt the solution of modern traffic problems, must consider all factors of the situation. He may doubt the wisdom of widening as a solution. He certainly can foresee that there are limits of the extent to which widening may be carried out. He must think about the feasibility of decentralization and its economic consequences. Granted that confiscation might expedite the widening programs, the engineer realizes that

this might not be unmitigated good. There are limits beyond which almost any developmental change cannot be made to economic advantage. Congestion in crowded districts is already leading in a natural way to the development of new business areas in many cities. Therefore, engineers who are aware of their social responsibilities should not be misled into developing an enthusiasm for a point of view which apparently considers only one phase of the problem. The necessity for confiscation is predicated upon the fact that taxation and the management of municipal units (where the bulk of the burden would fall) are such that the resources for compensation under condemnation are not available. Possibilities for the exercise of sovereignty by means of taxation are without limit, except that taxes be made so great as to absorb all private values in property. A remedy for the municipal financial situation might be suggested as a solution of the problem under discussion.

It seems that "in the long run" the practice of condemnation would prove to be more equitable. In effect, this practice involves an insurance element. All citizens unite through the taxation system and agree to compensate those who have located where society later decides they are in the way. It would seem that an extension of the idea of compensating those who are caused to suffer loss for the public good by fire, water, or disease might be in the direction of justice. The question of accrued values occurring as a result of widening is another and broader one. Undoubtedly, many properties adjacent to widening appreciate in value without suffering any loss. The entire question of rights in property is involved. An unlimited extension of the idea of confiscation would lead to a reformulation of the principles which guarantee security in the possession and use of property. This unlimited extension of confiscation may be as logical a step as that from quarantine to street widening.

It is necessary to stop and consider desirability by making an estimate of effect. When land is taken for street widening there is a loss of that land for other uses. The question is whether that loss shall be borne by the freeholder or by the social group that needs the wider street. Confiscation would not remove the financial problems; it would merely shift the burden. Faulty taxation or abuses under existing practice do not necessarily argue for confiscation. Perhaps good arguments can be given, but it would seem that the author of this paper has not presented them. Until the effect of the proposed solution is injected into the argument and until engineers are convinced that all other possible solutions are by comparison not as good in effect, it should not be expected that they will respond to the appeal made to them.

DONALD M. BAKER,[13] M. AM. SOC. C. E. (by letter).[13a]—The theory advanced by Mr. Williams is interesting, but if it were generally adopted it might establish a dangerous precedent; and, through an extension of the same theory to similar subjects, it might lead to rather alarming consequences. The theory advocated is that narrow streets in the present age of intensive operation of motor-driven vehicles are dangerous to public health and safety

[13] Cons. Engr., Los Angeles, Calif.

[13a] Received by the Secretary, March 28, 1931.

and that the governing agency should be able, by giving proper notice to the owners of abutting property of its intention, to take sufficient of this abutting property, at some future date, to make a highway of a width adequate to satisfy traffic demands, this taking to be without compensation to the owners.

Narrow streets in themselves are not dangerous to public safety. On the contrary, due to their very narrowness, traffic is deterred from using them. Wider streets attract more and faster traffic; in fact, they invite traffic of a nature which is more likely to become dangerous to public safety through accidents than that occurring on narrow streets. As to public health, there are data pointing to the fact that on narrow streets lined with tall buildings the health of the users thereof, and particularly that of the occupants of buildings on adjoining property, may be somewhat affected by carbon monoxide (CO) given off from the exhausts of automobiles. Evidence on this point, however, is not conclusive. Narrow streets do cause a clogging of vehicular circulation, but this affects primarily the values of the adjacent property, and it alone suffers. Without adequate opportunities for circulation, activity moves elsewhere and values depreciate.

If the principle advocated by the author were generally adopted by the Courts it might lead to a wholesale taking of property for public purposes without compensation. Consider, for example, the case of a badly congested neighborhood where there is urgent need for a playground. Such need might be just as severe from the standpoint of public health and safety as the need for wider streets. Under the principle advanced, the governmental agency could state that at a date five or ten years in the future it proposed under its sovereign powers to take property necessary for this playground from the owners thereof, without compensation. In the meantime, the owners must adjust themselves to the inevitable, being warned that any improvements or alterations made upon the property would be made at their own risk. It would be a simple matter in time to extend this principle to the acquisition of all property needed for public purposes.

The illustrations of the exercise of sovereignty, cited by the author, are emergency cases for which no other remedy is available. When buildings in the path of a fire are dynamited to stop the approach of a widespread conflagration, in all probability they would have been consumed by the fire in any event. The exercise of the right of quarantine is the only present feasible way of stopping an epidemic. It is an emergency measure. Property as represented by gambling devices may be seized because, in general, such devices are owned in violation of the law, and the owner may not come into Court with clean hands to reclaim them. The extermination of nuisances falls in a different category. If the nuisance existed before the surrounding development was made, the promoters of this later development would be forewarned of it and, in some cases, would have no remedy. If the nuisance commenced after the surrounding area was developed, the party causing the nuisance was forewarned and instituted it at his own peril.

The only reason for the establishment of set-back lines for future street widening is that it would be uneconomic to make the widening at the time the

set-back lines were proposed. Either traffic needs are not sufficient at present to require it, or its cost would be such that the property assessed for the widening, be it a district or the entire city, could not afford to pay the resulting price. In other words, the benefits resulting from the widening would not equal the cost thereof. In the first instance, it is a perfectly proper procedure, which should be supported by the Courts, to establish lines beyond which no owner may build and expect to obtain future damages for the destruction of that portion of his building extending beyond the line. This procedure is a wise one and prevents the public from being subjected to excessive and unwarranted damages when widening occurs. However, if the public is not in a position to pay for the land at the time the set-back lines are established, the owners should be compensated for the land taken at such time as the project becomes feasible and economical.

The writer also disagrees with Mr. Williams as to the general enhancement in values which result from a street widening or opening. It is true that such enhancement in value does occur in some instances where a portion of a parcel of property has been condemned, but this is rather the exception than the general rule. In many cases the widening and improving of a street with the resultant increase in traffic causes an actual detriment to the owners of abutting property, with the benefits, if any, accruing to property on side streets, which streets achieve greater accessibility without being injured by increased traffic.

City planners have established through investigation a definite relationship between business frontage and supporting population. With the present density of motor-vehicle traffic it is absolutely impossible to fill up the frontage on all the major traffic arteries with business. It is only in locations where business is justified and warranted that a resulting enhancement in values will occur. The widening of a quiet residential street into a heavily traveled artery will ruin such street for single-family residential use unless the property abutting thereon is developed into rather large holdings where the residences can be well set back from the property line, masked by shrubbery, hedges, etc., and thus screened from resulting confusion, noise, and dust.

Furthermore, such traffic arteries are not desirable for multiple residential or apartment-house purposes. Resulting noise, dust, etc., occasioned by heavy traffic makes the construction of these buildings—unless they too can be screened effectively from the traffic—an economic impossibility. The general effect of an extensive street-widening and improvement program, taking into consideration all the property involved, is a "freezing" of a considerable proportion of the frontage thereon. It cannot all be used for commercial purposes. In many cases, it is unsuitable for multiple residential or apartment structures, and entirely unsuitable for single-family residential uses. This has been the actual experience in Los Angeles, Calif., during the period from 1927 to 1930.

There is no question but that traffic congestion caused by the rapid increase of motor vehicles requires drastic remedies, but these remedies lie more along

the lines of proper planning and budgeting of the program of street widening, proper methods to be developed for the allocation of costs, more aggressive opposition on the part of city authorities toward unduly high awards in condemnation proceedings, and greater realization on the part of the Courts that excessive awards will prevent the ultimate achievement of a satisfactory and necessary arterial traffic plan, rather than in the adoption of the principle suggested by the author. Acquirement of adequate traffic capacity solely at the expense of abutting property is not just, because usually not the abutting property, but the entire surrounding district, is benefited, and the district should contribute toward the benefits received. If these benefits do not justify the cost, then the project is uneconomical and should not be attempted.

E. G. WALKER,[14] M. AM. SOC. C. E. (by letter).[14a]—The new idea offered in this paper involves rather drastic action. There can be no question of the correctness of the basis upon which the author ultimately relies; namely, that, in an emergency, the community as a whole has the right to interfere with private rights and to upset them completely if necessary in the interests of the people. However, in stretching his analogies of fire-fighting and quarantine to make them the bases of proposals for a drastic alteration of the existing principles of dealing with street widening, the author is not completely logical.

Conflagrations that make it necessary to enter private land and destroy buildings and other property thereon, and epidemics that require the isolation of all individuals coming from an infected area, are both subjects which are but partly under human control. Drastic curtailment of individual liberty entailed by remedial measures to which the author refers in his paper are brought about for that reason.

On the other hand, the regulation of traffic is under the entire control of the authorities appointed to deal with it. There is, therefore, no more reason for expropriating private property in one place than there is in another. If a particular road becomes too congested the traffic can be diverted to another one, or new roads can be built through less congested areas to by-pass traffic which is not proceeding to a destination in the area. The logical outcome of the author's suggestion is that any land required for road development could be expropriated without compensation. It would then follow that any property required for any purpose—which any State or local government that happened to be in power at the moment might consider to be in the interests of the community—could also be expropriated without compensation.

There is certainly much to be said against the huge and frequently inflated values which are assessed upon property taken for street widening in congested areas. There is room for considerable improvement in making the assessments. The argument that has been advanced frequently, is that these values belong in great part to the community because, but for the activity of the community, central sites would not reach such high values. Attempts have been made in the past to appropriate to the community such proportion

[14] (Maxted & Knott), London, England.

[14a] Received by the Secretary, April 1, 1931.

of the increase of value of property sites as could be claimed to be due to general communal development. These have failed primarily because of the impossibility of assessing any value for a site independently of the existence and activities of the occupants of surrounding sites.

The problem of taking land for road widening is full of difficulties and can be solved only by having reasonable regard to the interests of the individual as well as to those of the community as a whole. The suggestion that property should be taken without any compensation whatsoever is much too drastic. It is as unfair in one direction as the influences which frequently are brought to bear to enhance values for compensation, are in the other direction.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SURVEYS FOR SWIFT RIVER RESERVOIR OF THE BOSTON METROPOLITAN WATER SUPPLY

Discussion

By Messrs. C. B. Breed, and C. L. Hall

C. B. Breed,[4] M. Am. Soc. C. E.—This paper describes a surveying project unique in its performance, although quite common in its general aspects. It involves procuring a great deal of detail, the accuracy of which is well controlled where needed, yet "sketchy", where a low degree of precision is sufficient. It is a problem in surveying economics. In such problems good judgment based upon considerable experience is required in order that the survey-dollar may be reflected properly in the resulting data or plans. It is very easy to spend too much money on unimportant parts of the survey and too little on essentials. In fact, the opinion of well qualified surveyors may properly differ as to the most practical assembly of surveying methods to apply in order to reach a balanced economic result.

While the opinion of the speaker differs somewhat from that of the author in a few instances, it should be borne in mind that Mr. Hammond was on the site and that the speaker has not been on this project; the author has seen conditions at first hand while the speaker has viewed them through Mr. Hammond's lucid description of the conditions he encountered.

The control of the entire survey appears to have been logically laid out. It was properly joined with the triangulation stations established by the previous Massachusetts State Commissions. Mr. Hammond's comment that many valuable data of this kind are available and in usable form cannot be emphasized too forcefully. Many surveyors would gladly make use of such data, but, unfortunately, private projects too often have to be started before the surveying party has had time to connect it with the Government base of levels or triangulation control. In State work of the character described by Mr. Hammond, it is more readily done than in private projects, because more time is usually permitted for preliminaries.

Accuracy of Instrument Work.—It is somewhat surprising to learn that a 1′ transit was used in the triangulation system. The idea of using a 10″

Note.—The paper by N. LeR. Hammond, Esq., was presented at the meeting of the Surveying and Mapping Division, Boston, Mass., October 10, 1929, and published in February, 1931, *Proceedings*. This discussion is published in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

[4] Prof. of Railway and Highway Eng., Mass. Inst. Tech.; Cons. Engr., Boston, Mass.

transit may be abandoned because of its weight, but a 30″ or a 20″ transit of the same weight as a 1′ instrument may readily be obtained. By using a 20″ transit, the adopted precision could have been reached more quickly; or, with the same number of repetitions of angles, a higher precision could have been reached for the same cost.

It is so easy to obtain "closures" of much less than 8″ that it seems a pity that such a well-planned system as that described by the author, could not have been surveyed with greater precision. The time required for reconnaissance and signal building is the same in either case. The additional few minutes required to obtain the angle within 2 or 3″ is negligible. Had this been done, the results obviously would be of greater value to future surveys in that general district.

It seems a little unusual to measure the base lines with an ordinary steel tape and to determine and apply temperature corrections throughout the work, when an invar tape could have been used and the temperature corrections omitted. Of course, the invar tape is more brittle and requires more care in handling in rough country. It seems doubtful, however, whether lines measured by the ordinary tape as described would give a better base than could have been obtained by using the old State triangulation distances or by computing bases from the geographical position of those old State monuments which were properly joined with the system.

The application of the plane co-ordinate system for the location of all points is especially adapted to the ready replacing of lost points in a district where so much cutting of timber and general change in immediate surface of the terrain is to be made.

Transit and Stadia Versus Plane-Table.—While the control of the topographical map by triangulation and by precise levels is excellent, the speaker questions the advisability of using transit and stadia for detail topography. In the open country the plane-table has distinct advantages over the stadia and transit methods. Even in wooded countries, the speaker prefers the plane-table for economy and for other reasons.

The number of points that have to be located is much smaller; the shape of the terrain ought always to be sketched while it is before the eyes, because no amount of experience in office plotting and in drawing contours based on many plotted points in the office can represent so faithfully the shape of the ground, as the work of a good topographer who sketches the contours in the field and governs their accuracy by the position and elevation of a few intelligently selected points; and, lastly, the cost is less for a given accuracy of plan.

The Cruising and Sketching Method.—In dense woods, such as those described by Mr. Hammond, the speaker has found that the following cruising and sketching method gives satisfactory results for much of the topographic work of the character he has had to produce. Stadia lines are run by plane-table and alidate through clearings in the woods at intervals of, say, 500 to 5 000 ft., preferably parallel to each other; and contours are sketched on either side of these cleared lines for a short distance into the dense woods. The topographer then "cruises" the country between these stadia lines by running compass lines (usually at right angles to the stadia lines) at, say, 100 to 300-ft. intervals.

He starts from one stadia line with the elevation determined by stadia level and runs at right angles through the woods to the other line. He sketches contours on a small hand-sketching board as he goes, using a hand level or a clinometer for determining elevations and for checking these elevations at the other stadia line ahead. Then he returns on the next range to the first stadia line, obtaining all the topography over the route which he has "cruised".

Those data are transcribed directly on to the plane-table sheet which has been left on the first cleared stadia line. He then proceeds to the next pair of ranges, running south, say, on one and back north on the other, checking out at each time on his levels, on his distance, and on his position up and down on the stadia line as he reaches the clearing where that line is run. This method gives an independent control on all the topography. It provides a check on all the cruising work, and it is the most economical method of which the speaker knows for obtaining fairly accurate contours in heavily wooded districts. The transcription of the contours drawn on the sketching board on to the plane-table discloses any apparent error, which should be corrected immediately in the field.

Cemetery Surveys.—The survey of cemeteries also can be readily accomplished by the plane-table, using taped measurements, in the following economical manner. First, select a system of triangulation, the vertices of the triangles being the outer corners of stones marking cemetery lot corners. These triangles can be, say, 400 to 800 ft. on a side. Measure these distances with a tape. Plot them on the plane-table sheets by use of the beam compass and scale. Use these points for the control of the detailed survey to follow.

The plane-table is set up at the corner of one of these stone bounds and oriented by sighting on another triangulation point. Several other existing, easily recognized, monuments are then cut in. The plane-table is next taken to another triangulation point and these same points are cut in and thus located on the plan. This does not require sending a man to the monument for the sight.

An iron rod three times the size of an ordinary lining pin is driven in the ground at the plane-table station. This rod is driven through the tape ring. The tapeman measures the distance from this pin to the various lot corner markers and the plane-table cuts in the direction with the alidade. He then scales on each line the distance called off by the tapeman, draws the boundary lines, and records the lot, its serial number, and the family name. Two men are all that is needed in this field party. They can survey a great number of lots in a day in this manner. It is well not to extend the measurements from the plane-table beyond, say, 80 to 100 ft. Then, the plane-table should be moved to another point which has been located from the last point. At any time the plane tabler can cut in his postion from the many monuments which appear on his sheet, and thus check his position. In fact, it is to his advantage when he can cut in three or four characteristic monuments within his sheet limits.

The next step is to measure with the tape the four sides of each lot and the two diagonals. These are recorded to the nearest 0.1 ft. on the plane-table sheet, the scale of which is 1 in. = 20 ft.

Office Work.—On a rainy day the individual lots are plotted on large cards forming a card catalog. The upper half of the card is cross-sectioned and the lower part is in lines and columns. In the upper part the lot is plotted to the scale of 4 ft. to the inch, or 8 ft. to the inch. These are plotted from the four dimensions and the two diagonals which were recorded in the field. In the lower portion of the card the tabulation of names, dates of birth and death, etc., are recorded. These are copied from the cemetery records.

The card is then taken into the field, and at each individual lot the headstones, footstones, and monuments are located by tape measurements and ranges. Each grave is identified from the list of interments in the lower portion of the card and from the wording of markers, and if there are interments that cannot be found that are indicated in the list, or graves that appear on the lot which are not in the list, further investigation is made to determine who the the person is that is buried at that spot or whether or not there is any interment.

The result of this kind of a survey is a plan of the entire cemetery on the scale of 1 in. = 20 ft. which, of course, can be reduced to any desired scale. It shows all the roads, paths, lots, and the water system. This is supplemented by a card catalog plan of every individual lot that shows every physical thing upon it. It records each interment by a number in the tabulation and by an actual drawing of the shape of the lot on the upper portion of the card, each interment being given a number corresponding to the number in the tabulation.

The advantages of this method are: (1) It is controlled by the measured triangulation, and a detail of every lot is drawn while that lot is before one's eyes; (2) it requires only two men in the field party; (3) it is economical; and (4) it presents plenty of work for rainy weather as well as for fair days.

The Aerial Survey.—In the property line surveys for real estate plans, aerial maps (or contact prints) were used on the Swift River surveys for two appropriate purposes, namely (*a*), reconnoissance; and (*b*), filling in, or identifying, details.

These contact prints form a good check on the position of physical features located by the stadia and transit notes when they are plotted in the office, in that the finished map should (except for distortion due to difference in elevation) appear like the contact prints in so far as fence lines and the location of buildings, roads, and other details of flat topography, are concerned. Of course, they do not form any check on contour locations except that where a steep face of a ledge may occur or a brook is located the contact prints should give valuable data.

In aerial work of this kind, the speaker has found it advantageous to place cheese-cloth markers at points which he wanted to identify readily in the contact prints, where the object was one that probably would not show up well on the print. These places can readily be marked by 3-ft. strips of cheese-cloth laid at right angles over these points, or in the form of a circle around the points. The little white crosses or circles show up clearly in the photographs.

Identifying Property Lines.—The analysis of evidence of property lines is an excellent practice. In wood lots these should properly show the position of blazes marking property limits; and, where different blazed lines are

encountered on the same general property line, the apparent age of the blazes should be determined by cutting into the trees and counting the annual rings, and these facts should be noted on the analysis so that the line adopted may be supported by the strongest evidence found.

While the edge of timber clearings, which usually show up clearly in contact prints, is some evidence of property lines, it should be supported by other evidence as a rule, because it is not uncommon to find that when timber has been cleared an encroachment on adjacent property has been carelessly made, for whch damage may or may not have been adjusted.

As stated at the beginning of this discussion, the problem is one of surveying economics. The speaker has tried to keep in mind the cost of the surveys and to suggest such modifications, as, in his judgment, would give either better results for a given outlay or would have given as accurate and complete results for less money.

C. L. Hall,[5] M. Am. Soc. C. E. (by letter).[5a]—It is noted that Mr. Hammond adopted the equivalent of the Army grid system of co-ordinates for his control, yet retained true azimuths. The writer's experience with the use of a system of rectilinear co-ordinates for surveys of small areas has led him to believe that it is much simpler to use grid azimuth when rectilinear co-ordinates are available. In this system, the true azimuth of any line at the point of tangency of the plane of projection becomes the grid azimuth for all lines drawn parallel thereto everywhere on the map; and hence the angle which any straight line on the earth's surface makes with grid north is constant for the entire length of the line. It is very little trouble to calculate grid azimuths between points the rectilinear co-ordinates of which are known because the cosine of the angle between the line joining the two points, and grid north, can be determined at once by noting the proper relation between the differences of their x and y- co-ordinates, respectively.

No advantage appears to accrue from the use of true azimuths, except possibly that lines in legal documents can be more readily described in ordinary surveyor's language. However, no one expects exact accuracy in the azimuths of boundary lines. Admittedly, differences in true azimuth over an area 11 miles from east to west will not be great, but if no grid azimuth is established field surveyors will be unnecessarily annoyed whenever they run control traverses.

Under "Aerial Surveys," Mr. Hammond indicates that he could see no use in aerial photographs enlarged from 1:13 200 to 1:4 800. This agrees with the writer's experience. For general checking, an enlarged aerial photograph may sometimes be more convenient (although seldom more economical) than a pantographed enlargement; but as enlargement always reduces clarity, it will never be as satisfactory as the original print, in studying the country. If surveys are being made instrumentally on a scale of 1:2 400, neither an aerial photograph nor an enlargement thereof will appreciably reduce the cost of field topography, except as an aerial view permits better planning. The scale of the aerial view, thus used, is relatively unimportant.

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[5a] Received by the Secretary, February 27, 1931.